Exam 4   Calc 2   12/5/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find the sum of the series $\frac{3}{4}-\frac{3}{10}+\frac{3}{25}-\frac{6}{125}+\frac{12}{625}-\ldots$

2. Give a power series for $f(x)=\frac{\sin x}{x}$ of at least 4^{th} degree.

3. Find a Taylor polynomial of degree at least 4 for $f(x) = \cos x$ centered at $x = \pi/2$.

4. Determine whether $\sum_{n=1}^{\infty} \frac{3n}{n^3 + 1}$ converges or diverges.

5. Determine whether $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converges or diverges.

6. Find the radius of convergence of the Maclaurin series for $f(x) = e^x$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, this sucks. I can do every single homework problem from those sections about comparison tests and integral tests and everything, so I figured I was totally good. But then our review sheet had this really weird question about, like, if this one series a_n converges and this other series is, like, $b_n = a_n + 3$, then can that b_n series converge. I got nothin', 'cause it's the wrong way for the comparison stuff. So I guess doin' homework just isn't worth it, so I'm gonna go play some more World of Warcraft. "

   Help Biff by explaining what he might be able to do in an instance like this.

8. Use a Maclaurin polynomial of at least 7th degree to approximate $\int_0^{0.1} \frac{1}{1+x^3}\,dx$ to 4 decimal places.

9. a) Determine whether $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ converges or diverges.

b) What is $\lim_{n \to \infty} \frac{n^n}{(2n)!}$?

10. The radius of convergence of the series $\sum_{n=1}^{\infty}(-1)^n \frac{3^n(x-4)^n}{n^2}$ is 1/3. Are the endpoints included?

Extra Credit (5 points possible): Prove that if the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then the series $\sum_{n=1}^{\infty}\left(\frac{n+1}{n}\right)a_n$ is also absolutely convergent. [Stewart 5th p. 787]