Exam 1 Calc 3 9/28/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to y.

2. Find an equation for the plane tangent to $z = x^2y + y$ at the point (3,–2).

3. Show that $\lim_{(x,y)\to(0,0)} \dfrac{xy - y^2}{x^2 + y^2}$ does not exist.

4. Write the appropriate version of the chain rule for $\dfrac{\partial u}{\partial t}$ in the case where $u = f(x, y)$, $x = x(r, s, t)$, and $y = y(r, s, t)$. Make clear distinction between derivatives and partial derivatives.

5. Let $f(x,y) = y \ln x$. Find the maximum rate of change of f at the point (2,4) and the direction in which it occurs.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this Calc 3 stuff is killing me. There was this problem on our exam, and I did everything right except it was like a trick question, because the prof asked for the rate of change in this one direction, right? And I knew that meant a directional derivative, and I like them 'cause it's just a formula, right? But it was unfair because the direction he gave us wasn't a unit vector, and I forgot to make it one. So since it's multiple choice, I got zero credit, even when it was something stupid like that. Why would it have to be a unit vector anyway? I mean, it's just s'posed to be a direction, right, and the direction's the same."

   Explain clearly to Biff why it matters to use a unit vector in finding directional derivatives.

8. Find the maximum and minimum values, in the form (x, y, z), of the function $f(x, y) = x^2y$ subject to the constraint $x^2 + y^2 = 36$.

9. You're in charge of a space mission to bring back soil samples from Mars, which may or may not bring contaminants which will destroy all life on Earth. Part of your assignment is to find the largest (in volume) rectangular box that can be fit into the parabolic nosecone of the return module which will bring the samples back to Earth. The nosecone is shaped like the region between $z = 4 - x^2 - y^2$ and the plane $z = 0$ (where the units are in meters). What are the exact dimensions and volume of the largest box possible?

10. Consider the surface $f(x, y) = x^2 + 2xy + y^2 + 2x$. There is a collection of points all having the same directional derivatives (in any given direction) as the origin. Describe this collection.

Extra Credit (5 points possible):

Suppose that a point is traveling along the curve with parametric equations $x = \sin 2t$, $y = \cos t$ on the surface of the paraboloid $z = x^2 + y^2$. At which point along this path is the altitude of the point greatest?