Exam 3a   Calculus 3   11/20/2007

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F}(x,y,z) = y\,\mathbf{i} + (x - z)\,\mathbf{j} + (2 - y)\,\mathbf{k}$ for a line segment C from (0,3,–2) to (2,0,1).

2. Let $\mathbf{F}(x,y,z) = -y\,\mathbf{i} + x\,\mathbf{j} + a\,z\,\mathbf{k}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a counterclockwise (when viewed from above) circle of radius 2 centered on the z-axis in the plane $z = 5$.

3. Evaluate $\int_C x^4 dx + xydy$, where C is the triangular curve consisting of the line segments from (0,0) to (1,0), from (1,0) to (0,1), and from (0,1) to (0,0).

4. A really goofy cult leader has predicted that at midnight on Thanksgiving, every point within and around the Earth will suddenly begin radiating turkey-flavored jelly beans according to the vector field $\mathbf{F}(x,y,z) = 11x\ \mathbf{i} + 22y\ \mathbf{j} + 2007z\ \mathbf{k}$. Just in case this incredibly unlikely event should come to pass, compute the total number of jelly beans that would radiate outward through the surface of the Earth, i.e. the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where S is a sphere of radius approximately 4000 miles.

5. Compute the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = y\,\mathbf{i} - x\,\mathbf{j} - 2\,\mathbf{k}$ and S is the surface of the paraboloid $z = x^2 + y^2$ (with upward orientation) within the cylinder $x^2 + y^2 = 4$.

6. Show that for any function f(x,y,z) with continuous second partials, curl(grad f) = **0**. Make clear how you use the continuity condition.

7. Buffy says "Like, Calculus is *so* unfair. My professor gave us this test, and we were supposed to show that for this line integral thingy it didn't matter what way you went, like the path or whatever. So I like, did it for two different parama-whatevers, and, like, I got the same thing both ways, which is like a miracle for me anyway, but my professor wrote all this bad stuff that I totally don't understand about how that wasn't the right thing to do. She gave me almost no credit at all! But, my *God*, how else could I do it? I mean, it's not like I have enough time to do every way of connecting those two points, cause there's, like, *lots* of them."

   Explain (clearly enough for Buffy to understand) how such a thing can be done without actually trying an infinite number of paths, and tell her what's flawed about her approach.

8. Evaluate $\iint_S \text{curl}\,\mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = 2x^2\,\mathbf{i} + \mathbf{j} + xy\,\mathbf{k}$ and S is the surface of the paraboloid $z = x^2 + y^2$ (with upward orientation) within the cylinder $x^2 + y^2 = 4$.

9. Let $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$, and let S be the portion of a cylinder of radius R centered on the z-axis between $z = 0$ and $z = h$ for some positive constant h, with outward orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

10. Show that the surface area of a unit sphere (with parametrization $x(u,v) = \sin u \cos v$, $y(u,v) = \sin u \sin v$, $z(u,v) = \cos u$) is 4π.

Extra Credit (5 points possible): Find the positively oriented simple closed curve C for which the value of the line integral $\int_C \left(y^3 - y\right)dx - 2x^3 dy$ is a maximum.