Quiz 5 Calculus 3 11/5/2007

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F}(x,y,z) = \langle -y, x\rangle$ and C a circle of radius 2 centered at the origin and traversed counterclockwise.

I. $\vec{r}(t) = \langle 2\cos t, 2\sin t\rangle$ for $0 \le t \le 2\pi$

II. $\vec{F}(\vec{r}(t)) = \langle -2\sin t, 2\cos t\rangle$

III. $\vec{r}\,'(t) = \langle -2\sin t, 2\cos t\rangle$

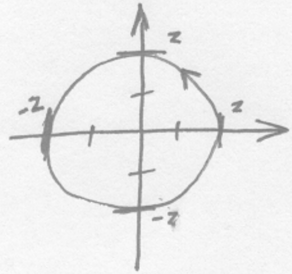
IV. $\int_0^{2\pi} \langle -2\sin t, 2\cos t\rangle \cdot \langle -2\sin t, 2\cos t\rangle\, dt$

V. $\int_0^{2\pi} (4\sin^2 t + 4\cos^2 t)\,dt$

$= \int_0^{2\pi} 4\,dt$

$= 4t\Big|_0^{2\pi}$

$= 8\pi$



2. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F}(x,y,z) = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy + 2z)\,\mathbf{k}$ and C a line segment from $(1,0,-2)$ to $(4,6,3)$.

Is there a potential function? Yes: $xyz + z^2 = f(x,y)$

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so $\int_C \vec{F}\cdot d\vec{r} = \int \nabla f \cdot d\vec{r} = f(4,6,3) - f(1,0,-2)$

$= (4\cdot 6\cdot 3 + 3^2) - (1\cdot 0\cdot -2 + (-2)^2) = 81 - 4 = \boxed{77}$

Yes!

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