Quiz 6 Calculus 3 11/9/2007

Each problem is worth 5 points. Clear and complete justification is required for full credit.

Let $\mathbf{F}(x,y,z) = \dfrac{x}{x^2+y^2+z^2}\mathbf{i} + \dfrac{y}{x^2+y^2+z^2}\mathbf{j} + \dfrac{z}{x^2+y^2+z^2}\mathbf{k}$.

1. Find div **F**.

$$\operatorname{div}\vec{F} = \frac{1(x^2+y^2+z^2)-x(2x)}{(x^2+y^2+z^2)^2} + \frac{1(x^2+y^2+z^2)-y(2y)}{(x^2+y^2+z^2)^2} + \frac{1(x^2+y^2+z^2)-z(2z)}{(x^2+y^2+z^2)^2}$$

$$= \frac{3x^2+3y^2+3z^2-2x^2-2y^2-2z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{1}{x^2+y^2+z^2}$$

2. Find curl **F**.

$$\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2+y^2+z^2} & \frac{y}{x^2+y^2+z^2} & \frac{z}{x^2+y^2+z^2} \end{vmatrix}$$

$$= \left\langle \frac{0(x^2+y^2+z^2)-z(2y)}{(x^2+y^2+z^2)^2} - \frac{0(x^2+y^2+z^2)-y(2z)}{(x^2+y^2+z^2)^2}, \right.$$

$$\frac{0(x^2+y^2+z^2)-x(2z)}{(x^2+y^2+z^2)^2} - \frac{0(x^2+y^2+z^2)-z(2x)}{(x^2+y^2+z^2)^2},$$

$$\left. \frac{0(x^2+y^2+\cancel{z}^2)-z(2x)}{(x^2+y^2+z^2)^2} - \frac{0(x^2+y^2+z^2)-x(2z)}{(x^2+y^2+z^2)^2} \right\rangle$$

$$= \langle 0, 0, 0 \rangle$$