Exam 2b Calc 3 10/31/2008

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Set up a double integral for the volume of the solid lying under the elliptic paraboloid $x^2/4 + y^2/9 + z = 1$ and above the triangle $R = [-1, 1] \times [-2, 2]$.

2. Set up a triple integral for the first-octant volume below $z = xy$ inside a cylinder with radius 5 centered on the z-axis.

3. Set up an iterated integral for the surface area of the portion of the parabolic cylinder $z = y^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

4. Set up iterated integrals for the x coordinate of the center of mass of the triangular region with vertices (0,0), (3,0), and (5,2), given that its density at each point is proportional to the distance of that point from the y-axis.

5. Suppose that a lamp has a bulb with a mean lifetime $\mu = 1000$ hours, which can be modeled with an exponential density function, and that as soon as one bulb burns out a second bulb replaces it. Set up an iterated integral for the probability that both bulbs burn out within a total of 2000 hours.

6. Find the Jacobian for the transformation $x = uv$, $y = vw$, $z = uw$.

7. Biff is a calculus student from Enormous State University, and he has a question. Biff says "So, these double integrals are killin' me. On our quiz there was this one where I got it wrong and the TA said something about how I did it too much, like I did it for a rectangle, but it was supposed to be for a triangle. So then on the exam, I divided my answers by two, 'cause a triangle is half of a rectangle, right? But then they marked those wrong too. So what's up with that?"

   Explain to Biff why his plan to divide by two for triangular regions does or doesn't work.

8. Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} 5\,dz\,dy\,dx$.

9. Set up iterated integrals for the x coordinate of the center of mass of the region bounded by the cone $z = \sqrt{x^2 + y^2}$, the sphere $x^2 + y^2 + z^2 = 9$, and with $x \geq 0$.

10. Set up an iterated integral (or integrals) for the volume of the region bounded between the surfaces $x^2 + y^2 = 9$ and $(y - 4)^2 + z^2 = 4$.

Extra Credit [up to 5 points possible]:
Jon likes mathematical birthday cakes. Suppose that next year his cake is shaped like the region bounded by the cylinder $x^2 + y^2 = 25$ between $z = 0$ and $z = 4 + y/2$. He plans to eat half of his cake on his birthday, and save the other half for the following day. If he cuts it in half by first cutting along the

ray where ? = 0, and then along the ray ? = x, what should x be?