Quiz 5a **Calculus 3** **11/12/2008**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

Let $\mathbf{A}(x, y, z) = \left\langle \dfrac{-z}{x^2 + y^2 + z^2}, 0, \dfrac{x}{x^2 + y^2 + z^2} \right\rangle$.

1. Compute div **A**.

$$\operatorname{div} A = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$= \frac{0(x^2+y^2+z^2) - (-z)(2x)}{(x^2+y^2+z^2)^2} + 0 + \frac{0 - x(2z)}{(x^2+y^2+z^2)^2}$$

$$= \frac{2xz - 2xz}{(x^2+y^2+z^2)^2} = 0$$

Nice!

2. Compute curl **A**.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{-z}{x^2+y^2+z^2} & 0 & \frac{x}{x^2+y^2+z^2} \end{vmatrix} = \left(\frac{0\cdot(x^2+y^2+z^2) - x\cdot 2y}{(x^2+y^2+z^2)^2}\vec{i} + \frac{-1(x^2+y^2+z^2) - -z\cdot 2z}{(x^2+y^2+z^2)^2}\vec{j} + 0\vec{k} \right) - \left(0\vec{i} + \frac{1\cdot(x^2+y^2+z^2) - x\cdot 2x}{(x^2+y^2+z^2)^2}\vec{j} + \frac{0\cdot(x^2+y^2+z^2) - -z\cdot 2y}{(x^2+y^2+z^2)^2}\vec{k} \right)$$

$$= \left\langle \frac{-2xy}{(x^2+y^2+z^2)^2}, \frac{2z^2 + 2x^2 - 2x^2 - 2y^2 - 2z^2}{(x^2+y^2+z^2)^2}, \frac{-2yz}{(x^2+y^2+z^2)^2} \right\rangle$$

$$= \frac{1}{(x^2+y^2+z^2)^2} \left\langle -2xy, -2y^2, -2yz \right\rangle$$