

Each problem is worth 5 points. Clear and complete justification is required for full credit.

$$\text{Let } \mathbf{A}(x, y, z) = \left\langle \frac{x}{x^2 + y^2}, \frac{y}{x^2 + y^2}, 0 \right\rangle.$$

1. Compute $\text{div } \mathbf{A}$.

dot product

$$\text{div } \mathbf{A} = \frac{(x^2 + y^2)(1) - x(2x)}{(x^2 + y^2)^2} + \frac{(x^2 + y^2)(1) - y(2y)}{(x^2 + y^2)^2} + 0$$

Great

$$= \frac{x^2 + y^2 - 2x^2 + x^2 + y^2 - 2y^2}{(x^2 + y^2)^2} = \frac{2x^2 + 2y^2 - 2x^2 - 2y^2}{(x^2 + y^2)^2} = \underline{\underline{0}}$$

2. Compute $\text{curl } \mathbf{A}$.

cross product

$$\text{curl } \mathbf{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2 + y^2} & \frac{y}{x^2 + y^2} & 0 \end{vmatrix} = \underline{\underline{\left(0\vec{i} + 0\vec{j} + \frac{(x^2 + y^2)(0) - y(2x)}{(x^2 + y^2)^2} \vec{k} \right)}} = \underline{\underline{\left(0\vec{i} + 0\vec{j} + \frac{(x^2 + y^2)(0) - x(2y)}{(x^2 + y^2)^2} \vec{k} \right)}}$$

Excellent!

$$= \langle 0, 0, 0 \rangle$$

$$= \underline{\underline{\vec{0}}}$$