Problem Set 7 Real Analysis Due 10/26/2008

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Prove that if a set $E \subseteq \mathbb{R}$ is closed, then $\mathbb{R} - E$ is open.

2. Prove that if a set $E \subseteq \mathbb{R}$ is open, then $\mathbb{R} - E$ is closed.

3. Prove that if f is continuous at $x = c \in [a, b]$, then for any sequence $\{x_n\}$ in $[a, b]$ converging to c, the sequence $\{f(x_n)\}$ converges to $f(c)$. [§3.1 #6k]

4. Suppose that S is a nonempty set of real numbers. Prove that $M = \sup S$ if and only if M is an upper bound of S and there exists a sequence $\{a_n\}$ in S with $\lim_{n\to\infty} a_n = M$. [§2.5 #5]

5. Give an example of a function f defined on a closed and bounded set D which is not bounded on D.

6. Give an example of a continuous function f defined on a closed set D which is not bounded on D.

7. Give an example of a continuous function f defined on a bounded interval D which is not bounded on D.

8. Give an example of a function f defined on a closed and bounded set D which does not attain a maximum value on D.

9. Give an example of a continuous function f defined on a closed set D which does not attain a maximum value on D.

10. Give an example of a continuous function f defined on a bounded interval D which does not attain a maximum value on D.