Problem Set 4 Calculus 2 Due 11/4/09

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 5 points. For full credit indicate clearly how you reached your answer. All work must be legible and submitted on clean paper without ragged edges.

1. A population growing according to the differential equation $\frac{dp}{dt} = k \cdot p \cdot (C - p)$ is said to be growing logistically. Find a general solution to this differential equation.

2. On the Island of Komodo[1] there live three species: reptiles, mammals, and plants. We will represent the numbers of these species, in thousands, a *R*, *M*, and *P* respectively. The following differential equation give the rates of growth (where *t* is in years) of reptiles, mammals, and plants of the island:

$$\frac{dR}{dt} = -0.2R - 0.04RM + 0.0008RP$$

$$\frac{dM}{dt} = -0.1M + 0.01RM$$

$$\frac{dP}{dt} = 2P - 0.002P^2 - 0.1RP$$

If the populations start at $R = 7$, $M = 4$, and $P = 600$, use Euler's Method (with $\Delta t = 2$) to estimate the populations at $t = 10$.

3. a) Who is eating whom on Komodo? Describe the nature of the interaction between each species.
   b) Describe what would happen to each of the populations on Komodo if the other two species were not present (i.e., what would happen to the Reptiles if there were no Mammals or Plants, etc.). You may want to use technical terms such as "exponential growth", "logistic growth", or "dying like flies".

4. a) An *equilibrium point* of a differential equation or system of differential equations is a value which makes the rate of change zero. Find all equilibrium points of the Komodo system.

   b) The Mammals on Komodo begin drinking energy drinks, which makes them much fiercer predators but without the attention span to finish a meal. This changes the differential equation for the Reptiles to:

$$\frac{dR}{dt} = -0.2R - 0.08RM + 0.0008RP$$

   What does this do to the equilibrium points?

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[1]This material due to or inspired by the *Instructors' Manual* from Hughes-Hallett, Gleason, et al.