Reference Sheet: Series Tests Calculus 2 11/17/2009

The Geometric Series Test: If a series is of the form $\sum_{n=1}^{\infty} a \cdot r^{n-1}$, then the series converges $\left(\text{to } \dfrac{a}{1-r} \right)$ if and only if $|r| < 1$

The Integral Test: Suppose f(x) is a continuous, positive, decreasing function on [c, ∞) for some $c \geq 0$, with $a_n = \mathrm{f}(n)$ for all n,

- If $\int_c^\infty \mathrm{f}(x)\,dx$ converges, then Σa_n converges also.
- If $\int_c^\infty \mathrm{f}(x)\,dx$ diverges, then Σa_n diverges also.

The Comparison Test: If Σa_n and Σb_n are both series with their terms all positive, and

- $a_n \leq b_n$ with Σb_n convergent, then Σa_n converges also.
- $a_n \geq b_n$ with Σb_n divergent, then Σa_n diverges also.

The Limit Comparison Test: If Σa_n and Σb_n are both series with their terms all positive, and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

for some finite, positive number L, then either both series converge or both series diverge.

The Alternating Series Test: If Σa_n is a series for which

- the signs alternate, i.e. a_n and a_{n+1} have opposite signs for all n
- the sequence $\{|a_n|\}$ tends to zero, i.e. $\lim_{n\to\infty} |a_n| = 0$
- the sequence $\{|a_n|\}$ is decreasing, i.e. $|a_{n+1}| \leq |a_n|$ for all n

then the series converges.

The Ratio Test: If Σa_n is a series for which

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L$$

then

- if $L < 1$ then the series converges absolutely.
- if $L > 1$ (or if the limit diverges to $+\infty$) then the series diverges.