Exam 1   Calc 3   9/25/2009

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to y.

2. Show that $\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{x^2 + y^2}$ does not exist.

3. Find the directional derivative of $f(x, y) = 3x^2y - y^4$ at the point $(2,-1)$ in the direction of the vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.

4. Let $f(x,y) = x^3 + 3xy$. Find the direction in which f is increasing fastest at the point $(-3, 1)$, and the rate of increase in that direction.

5. Jon's daughter Anika has left her Duck Game on again, filling the house with a maddening and incessant "***Quack! Quack! Quack!***" noise. As the battery runs down, the motor also heats up and increases its resistance. Ohm's Law states that $V = IR$. At the moment when $R = 500.\ \Omega$ and $I = 0.080$ A, $dV/dt = -0.020$ V/s and $dR/dt = 0.030\ \Omega$/s. What is dI/dt at this instant? Give your answer correct to at least 6 digits after the decimal point.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this Calculus stuff is, like, *sooo* confusing. Like, I understood from before about positive derivatives means increasing and negative derivatives means decreasing, right? But so now I asked our TA if that was the same still, and then it doesn't make sense if f_x is positive but f_y is negative, how can it be both increasing and decreasing at the same time?"

   Explain clearly to Bunny how to bring her previous understanding of increasing and decreasing functions to bear on functions of two variables.

8. Find the extreme values of $f(x, y) = 3x^2 + y^2 - y$ on the disk $x^2 + y^2 \leq 1$.

9. a) Find an equation for the plane tangent to the sphere $x^2 + y^2 + z^2 = 25$ at the point (3, 0, 4).

b) Find an equation for the plane tangent to the sphere $x^2 + y^2 + z^2 = 25$ at the point (3, 4, 0).

10. Find the maximum volume of a rectangular box in the first octant with three faces lying in the coordinate planes and one vertex on the plane $x + y + 4z = 4$.

Extra Credit (5 points possible):

If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is to enclose the circle $x^2 + y^2 = 2y$, what values of a and b minimize the area of the ellipse? [Stewart Ch. 14 Problems Plus #7]