Exam 3 Calc 3 11/24/2009

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Parametrize and give bounds for the portion of the cylinder with radius 2 centered around the x-axis between $x = 0$ and $x = 5$.

2. Let $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy + 2z)\,\mathbf{k}$, and C be the line segment from (1,0,–2) to (4,6,3). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. Evaluate $\int_C y^3 dx - x^3 dy$, where C is the circle $x^2 + y^2 = 4$, oriented counterclockwise.

4. Let F(x, y, z) = $\langle 2xz, 3z, -z^2 \rangle$, and let S be the paraboloid $z = x^2 + y^2$ below $z = 9$, along with a disc of radius 3 in the plane $z = 9$ centered at (0,0,9), all with outward orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

5. Let $\mathbf{G}(x, y, z) = \langle 5x, -z, y \rangle$, and note that curl $\mathbf{G}(x, y, z) = \langle 2,0,0 \rangle$. Let S be the portion of $x^2 + y^2 + z^2 = 4$ above $z = 0$, with upward orientation. Evaluate $\iint_S \operatorname{curl} \mathbf{G} \cdot d\mathbf{S}$.

6. Show that for any vector field in $\mathbb{R}^3$ whose component functions have continuous second-order partial derivatives, div curl $\mathbf{F} = 0$. Make it clear why the requirement about continuity is important.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod. This stuff is so totally confusing. I thought math was where you, like, did problems and stuff, but now there's all this vocabulary stuff, and it's more like French than math or something, you know? So our professor was saying that we should understand why line integrals on closed paths in conservative vector fields are always zero, and I'm not even sure what those words are. Help!"

   Explain clearly to Bunny what the underlined terms mean and why the conclusion is valid.

8. Let C be a smooth curve given by the vector function $\mathbf{r}(t)$, $a \le t \le b$. Let f be a differentiable function of two variables whose gradient vector ∇f is continuous on C. Show that

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$$

9. Let $\mathbf{F}(x, y) = Q(x, y)\,\mathbf{j}$. Let C be the line segment from (c, b) to (d, b). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

10. Let $\mathbf{G}(x, y, z) = \langle 2\ , z, -y \rangle$, and let S be the portion of the plane $z = 1$ in the first quadrant inside the cylinder $x^2 + y^2 = 9$, with upward orientation. Evaluate $\iint_S \mathbf{G} \cdot d\mathbf{S}$.

Extra Credit (5 points possible):
Prove the identity curl(f **F**) = f curl **F** + (∇f) × **F**.