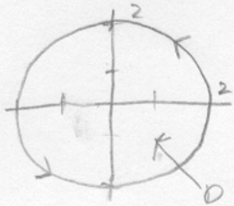
Quiz 10 **Calculus 3** **11/18/2009**

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\oint_C y^3 dx - x^3 dy$, where C is the circle $x^2 + y^2 = 4$.



$\vec{r}(t) = \langle 2\sin t, 2\cos t \rangle$

$0 \le t \le 2\pi$

By Green's Theorem:

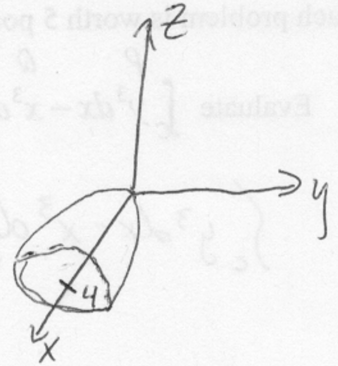
$\oint_C y^3 dx - x^3 dy = \iint_D (-3x^2 - 3y^2)\, dA =$

$-3\int_0^{2\pi}\int_0^2 r^2\, r\, dr\, d\theta = -3\int_0^{2\pi} \left[\frac{1}{4}r^4\right]_{r=0}^{r=2} d\theta =$

$-3\int_0^{2\pi} 4\, d\theta = \boxed{-24\pi}$

Good

2. Let $\mathbf{F}(x, y, z) = \langle 2,0,0\rangle$ and S be the portion of $x = y^2 + z^2$ behind $x = 4$, oriented in the direction of the positive x axis. Compute $\iint_S \mathbf{F}\cdot d\mathbf{S}$.



$x(u,v) = u^2+v^2$

$y(u,v) = u$

$z(u,v) = v$

$\bar{r}(u,v) = \langle u^2+v^2, u, v\rangle$

$\bar{F}(\bar{r}(u,v)) = \underline{\langle 2,0,0\rangle}$

$\bar{r}_u = \langle 2u, 1, 0\rangle$

$\bar{r}_v = \langle 2v, 0, 1\rangle$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2u & 1 & 0 \\ 2v & 0 & 1 \end{vmatrix} = \underline{\langle 1, -2u, -2v\rangle}$$

$$\langle 2,0,0\rangle\cdot\langle 1,-2u,-2v\rangle = 2+0+0 = 2$$

$$\underline{\int_0^{2\pi}\int_0^2 2r\,dr\,d\theta} = \int_0^{2\pi}\left[r^2\right]_0^2 d\theta$$

$$= \int_0^{2\pi} 4\,d\theta$$

Excellent!

$$= \left[4\theta\right]_0^{2\pi} = \boxed{8\pi}$$