Quiz 4 Calculus 3 Due 9/16/2009

This is an open-book, open-note, open-*Mathematica* take-home quiz. Each problem is worth 2 points. You are encouraged to work in groups of 2-4 and submit a single writeup for each group. You should include clear and complete, but not excessive, explanation of your conclusions.

1. Monday in class we computed the directional derivative of $g(x,y) = \cos x \cos y$ at the point (0,0) in a certain direction and found that it was 0. Victor noticed that even if we had changed our direction, we still would have found the directional derivative to be 0. Look at a graph of this surface and use it to understand why this happened. Identify a couple of other points on the surface where the same thing will happen, i.e. where the directional derivative will be 0 in every direction.

2. Look at a graph of the function $f(x,y) = \cos\left(\sqrt{x^2+y^2}\right)$ and compare it with the function $g(x,y) = e^{-0.1\sqrt{x^2+y^2}}\cos\left(\sqrt{x^2+y^2}\right)$. Describe the similarities and differences between these two surfaces.

3. Find approximate coordinates (accurate to at least the nearest hundredth) for the highest and lowest points on the surface $f(x,y) = \dfrac{x-y}{2x^2+8y^2+3}$. [Hint: Make *Mathematica* do the hard work. Start with Plot3D to get a general idea of where the extreme values are, and shift to ContourPlot to get a clearer idea, refining your domain to zoom in on the exact locations.]

4. Find the lowest point on the intersection of the surface $z = x^2 + y^2$ and the vertical plane $3x + y = 5$. [Hint: You're welcome to try getting an idea of what's going on with *Mathematica*, but it's not a great tool for the job this time. Probably the best plan is to solve the equation of the line for y, then substitute that into the equation for the paraboloid. Now you can use Calc 1 methods to find the minimum value of z as a function of x.]

5. Find the highest and lowest points on the surface $z = x\,y^2 - x^3$ within the rectangle $-2 \le x \le 1$, $-2 \le y \le 2$. [The surface is called a monkey saddle, which I find pretty amusing.]