Exam 3 Calc 3 12/2/2010

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Parametrize and give bounds for the portion of a cylinder centered on the x-axis with radius 5 between $x = 0$ and $x = 10$.

2. Let $\mathbf{F}(x, y, z) = 4x^3yz^2\,\mathbf{i} + (x^4z^2 - 3y^2)\,\mathbf{j} + 2x^4yz\,\mathbf{k}$, and C be the line segment from (–1,1,2) to (1,3,1). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. Let $\mathbf{F}(x, y, z) = \langle 2xz, 3z, -z^2\rangle$, and let S be the paraboloid $z = x^2 + y^2$ below $z = 9$, along with a disc of radius 3 in the plane $z = 9$ centered at (0,0,9), all with outward orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

4. Let $\mathbf{F}(x, y) = \langle -y/2, x/2 \rangle$. Let C be the path parametrized by $\mathbf{r}(t) = \langle a \cos t, b \sin t \rangle$ for values of t beginning at 0 and going to 2π. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

5. Let C be the top half of a circle with radius 2 centered at the origin, along with the line segment from (–2,0) to (2,0), all with counterclockwise orientation. Evaluate $\int_C \left\langle xy, 3y^2 + x^2 \right\rangle \cdot d\mathbf{r}$.

6. Show that for any function f of three variables that has continuous second-order partial derivatives, $\operatorname{curl}(\nabla f) = \mathbf{0}$. Make it clear why the requirement about continuity is important.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod. This stuff is *so* wrong. I thought math was where, like, the answers were numbers, right? But now we're s'posed to be able to do these with this Stokey Theorem, you know? And so the professor said there might be questions for, like, orienting boundaries? Like, he said we should be able to figure out if there was a cylinder centered on the z axis, like from some bottom z to some top z, and with the little normal arrow thingies pointing out, then what would its boundary curves be and which way they orient. I liked it better when the answers were things like square root 2, you know?"

   Explain clearly to Bunny what the boundary of the surface she describes should be, and why.

8. Let $\mathbf{F}(x, y, z) = \langle z - y, x, -x \rangle$, and let C be the circle $x^2 + y^2 = 4$ in the plane $z = 0$, oriented counterclockwise. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

9. Let $\mathbf{G}(x, y, z) = \langle 0,2,2 \rangle$ and let S be the portion of $x^2 + y^2 + z^2 = 4$ above $z = 0$, with upward orientation. Evaluate $\iint_S \mathbf{G} \cdot d\mathbf{S}$.

10. Suppose $\mathbf{F}(x, y) = \langle a, b\rangle$ is a constant vector field (so a and b are constant real numbers). Show that the line integral of $\mathbf{F}$ on a full circle of radius r centered at (h, k) is 0.

Extra Credit (5 points possible):
Apply Green's Theorem to the vector field $\mathbf{F}(x, y) = -g(x, y)\,\mathbf{i} + f(x, y)\,\mathbf{j}$, for some simple closed curve C with positive orientation. Comment on the significance of your result.