Problem Set 2 Real Analysis Due 9/12/2010

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let $a, b \in \mathbb{R}$. Show that $|a - b| < \varepsilon$ for all $\varepsilon > 0$ if and only if $a = b$.

2. Suppose that $\{a_n\}_{n=1}^{\infty}$ is a convergent sequence. Is $\{a_n\}_{n=3}^{\infty}$ convergent as well? Prove it or give a counterexample.

3. Suppose that $\{a_n\}_{n=2}^{\infty}$ is a convergent sequence. Is $\{a_n\}_{n=1}^{\infty}$ convergent as well? Prove it or give a counterexample.

4. Suppose that $\{a_n\}$ and $\{b_n\}$ are series such that $a_n = b_n$ for all even values of $n \in \mathbb{N}$, and suppose $\{a_n\}$ converges. Show that $\{b_n\}$ converges as well, or provide a counterexample.

5. Show that the sequence $\{a_n\}$ converges to 0 if and only if the sequence $\{|a_n|\}$ converges to 0.

6. Let $\{a_n\}$ be defined by $a_n = \dfrac{2n}{n+1}$ for $n \in \mathbb{N}$. Show that this sequence converges.

7. Prove Remark 2.1.8(f).

8. Give an example of a sequence a_n whose value is 7 for infinitely many values of n, but which does not converge to 7.