Exam 1b Calc 1 9/23/2011

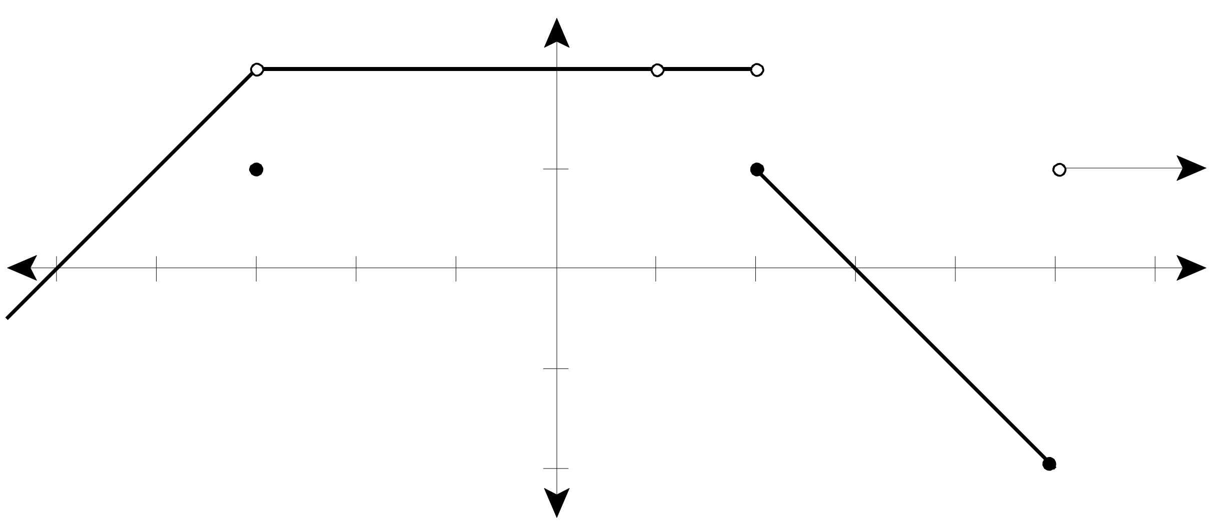
Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 through 3:

1. Find the following limits:
   a) $\lim_{x \to -3} g(x)$
   b) $\lim_{x \to 0} g(x)$
   c) $\lim_{x \to 2^-} g(x)$
   d) $\lim_{x \to 2^+} g(x)$
   e) $\lim_{x \to 2} g(x)$

2. For which values of x does the function fail to be continuous?

3. Find a slope function for $g(x)$.



4. Fill each blank below with a limit rule justifying that equality. [List of rules on last page]

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$$\lim_{x\to 2}\left(6x^2-5\right) = \lim_{x\to 2}\left(6x^2\right) - \lim_{x\to 2} 5$$

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$$= 6\lim_{x\to 2}\left(x^2\right) - \lim_{x\to 2} 5$$

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$$= 6\left(\lim_{x\to 2} x\right)^2 - \lim_{x\to 2} 5$$

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$$= 6\left(2\right)^2 - \lim_{x\to 2} 5$$

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$$= 6\cdot 4 - 5$$

$$= 24 - 5$$

$$= 19$$

5. Evaluate $\lim_{x \to 1^-} \dfrac{3x^2}{x^2 - 2x + 1}$.

6. Evaluate $\lim_{x \to \infty} \dfrac{3x^2}{x^2 - 2x + 1}$.

7. Evaluate $\lim_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. I did good in math in high school, but everything is totally different now. I mean, like, the one thing you knew before was that if there was a function with, like, a fraction, so like there was x minus 3 underneath, then there'd be one of those vertical asymptom things when x was 3, right? But we had this one on our test, and it was like $\dfrac{x^2-7x+6}{x^2-1}$, so it should have asymptoms when x is 1 and –1, right? But on our test I said that and they marked it wrong."

   Help Biff by explaining as clearly as you can why the graph of his function does not have the vertical asymptote he expected when $x = 1$, and what is in fact going on near that x value.

9. Evaluate $\lim\limits_{x \to a} \dfrac{x^4 - a^4}{x - a}$.

10. a) Determine whether $f(x)=\dfrac{|x-2|(x+1)}{x-2}$ is continuous where $x = 2$, and explain your reasoning clearly.

b) Is there any value of a for which the function $g(x)=\dfrac{|x-a|(x+1)}{x-a}$ will be continuous at every real number?

Extra Credit (5 points possible):

Evaluate $\lim\limits_{h\to 0}\dfrac{\sqrt[3]{8+h}-2}{h}$.

Algebraic Limit Properties **Calculus 1** **9/13/11**

Let a and c be constants. Then

Constant Rule for Limits: $\lim_{x \to a} c = c$

Rule X for Limits: $\lim_{x \to a} x = a$

And as long as $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ are real numbers,

Sum Rule for Limits: $\lim_{x \to a} \left[f(x) + g(x) \right] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

Difference Rule for Limits: $\lim_{x \to a} \left[f(x) - g(x) \right] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

Constant Multiple Rule for Limits: $\lim_{x \to a} \left[c \cdot f(x) \right] = c \cdot \lim_{x \to a} f(x)$

Product Rule for Limits: $\lim_{x \to a} \left[f(x) \cdot g(x) \right] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

Quotient Rule for Limits: $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ as long as $\lim_{x \to a} g(x) \neq 0$.

Power Rule for Limits: $\lim_{x \to a} \left[f(x) \right]^n = \left[\lim_{x \to a} f(x) \right]^n$