Exam 3b Calc 1 11/11/2011

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int \left(3x^5 + \sec^2 x - e^x\right) dx$.

2. Find all intervals on which $y = 2x^3 - 6x + 2$ is increasing.

3. Evaluate $\lim_{x \to \infty} \frac{x}{e^x}$.

4. Find the x-coordinates of the global maximum and minimum of $f(x) = x^4 - 4x^2 + 6$ on the interval [0,3].

5. For which values of x is $f(x) = \dfrac{\ln x}{x}$ concave up?

6. A rectangular storage container with an open top is to have a volume of 22 cubic meters. The length of its base is twice the width. Material for the base costs 12 dollars per square meter. Material for the sides costs 6 dollars per square meter. Find the cost of materials for the cheapest such container.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. They just keep making this calculus stuff harder, you know? I started out pretty good on this min and max stuff, but now they're saying there are gonna be true/false questions on the exam, so they can grade 'em all with a machine, and the samples they gave us were just crazy. Like, one was whether there could be a function that had two local mins with no local maxes. I can take the derivative and set it equal to zero, but I sure don't know how to tell anything if they don't give me a formula!"

   Help Biff by explaining whether the situation he describes might occur.

8. Let a and b be positive real numbers. Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$.

[Hint: Remember $(a^x)' = (\ln a)\, a^x$.]

9. Two supply centers are located at the points (0,1) and (0,–1). A manufacturing plant will be located at the point (4,0). Find the shortest collection of roads that connects these three points.

10. Suppose we have a function of the form $f(x) = x^3 + a\,x^2 + b\,x + c$. Are there values for the constants a, b, and c that allow the function to have a local minimum at (2,5) and local maximum at (–2,37)?

Extra Credit (5 points possible):

Show that $\lim_{r \to 0} \left(\frac{a^r + b^r + c^r}{3} \right)^{1/r} = \sqrt[3]{abc}$.