Quiz 2 Calculus 1 10/4/2011

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Let $f(x) = x^2 e^x$. Find an equation for the line tangent to $f(x)$ where $x = 1$.

$f'(x) = 2x \cdot e^x + x^2 \cdot e^x = e^x(2x + x^2)$

$f(1) = 1^2 e^1 = e$

$(1, e)$

$f'(1) = 3e$

$y - e = 3e(x-1)$

$y - e = 3xe - 3e$

$y = 3xe - 2e$

Excellent!

2. Let $g(x) = \dfrac{\sin x}{x}$. Find an equation for the line tangent to $g(x)$ where $x = \pi/2$.

Well, $g'(x) = \dfrac{(\sin x)' \cdot x - \sin x \cdot (x)'}{(x)^2}$ and $g\left(\frac{\pi}{2}\right) = \dfrac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)}$

$= \dfrac{x\cos x - \sin x}{x^2}$ $\qquad = \dfrac{1}{\frac{\pi}{2}}$

So $g'\left(\frac{\pi}{2}\right) = \dfrac{\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)^2}$ $\qquad = \dfrac{2}{\pi}$

$= \dfrac{0 - 1}{\frac{\pi^2}{4}}$

$= \dfrac{-4}{\pi^2}$

So the equation of our line through $\left(\frac{\pi}{2}, \frac{2}{\pi}\right)$ with slope $m = \frac{-4}{\pi^2}$ is:

$y - \left(\frac{2}{\pi}\right) = \left(\frac{-4}{\pi^2}\right)\left(x - \left(\frac{\pi}{2}\right)\right)$