Exam 1 Calc 3 9/30/2011

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to x.

2. Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

3. Write an appropriate version of the chain rule for $\frac{du}{dt}$ in the case where $u = f(x, y, z)$, $x = x(t)$, $y = y(t)$, and $z = z(t)$. Make clear distinction between derivatives and partial derivatives.

4. Let $g(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$. Find the directional derivative of g in the direction of $\langle 2, -1 \rangle$ at the point $(3, -4)$.

5. Let $f(x,y) = x^2/y$. Find the maximum rate of change of f at the point (2,3) and the direction in which it occurs.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this Calc 3 stuff is killing me. What's up with this directional stuff, anyway? There was this question on our review sheet for the exam, and it was weird, it was like, could there be anyplace on this one surface where the directional derivative was more than 1. I figured probably out of all the points and all the directions there must be, right? But this guy in my dorm said I had to use gradients, which sounds pretty crazy, 'cause the question wasn't even about gradients."

   Explain clearly to Biff how such a question might be approached, and why gradients might be relevant.

8. Find the maximum and minimum values (yes, the values, not just where they occur) of the function $f(x, y) = x + 2y$ subject to the constraint $x^2 + y^2 = 4$.

9. a) Let a and b be constants, and $f(x, y) = x^2 + ax + y^2 + by$. Find all critical points of this function and classify them as local maxima, local minima, or saddle points.

b) If we vary the function from part a to be $g(x, y) = x^2 + ax + y^2 + by + cxy$, where c is an additional constant, how does this change the collection of local extrema?

10. Show that if $f(x, y) = \dfrac{ax + by}{cx + dy}$, where a, b, c, and d are real numbers with $ad - bc = 0$, then $f_x = f_y = 0$ for all x and y in the domain of f [Briggs & Cochran, p.821].

Extra Credit (5 points possible):
Explain why the surface from #10 has the partial derivatives it does.