Exam 1 Calc 1 9/21/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 through 3:

1. Find the following limits:
   a) $\lim_{x\to 3^-} g(x) = \underline{0}$
   b) $\lim_{x\to 3^+} g(x) = \underline{0}$
   c) $\lim_{x\to 3} g(x) = \underline{0}$
   d) $\lim_{x\to 5^+} g(x) = \underline{1}$
   e) $\lim_{x\to 5} g(x)$ $\underline{\text{DNE}}$, $\lim_{x\to 5^-} g(x) = -2$ & that $\neq \lim_{x\to 5^+} g(x) = 1$

Good

2. For which values of x does the function fail to be continuous?

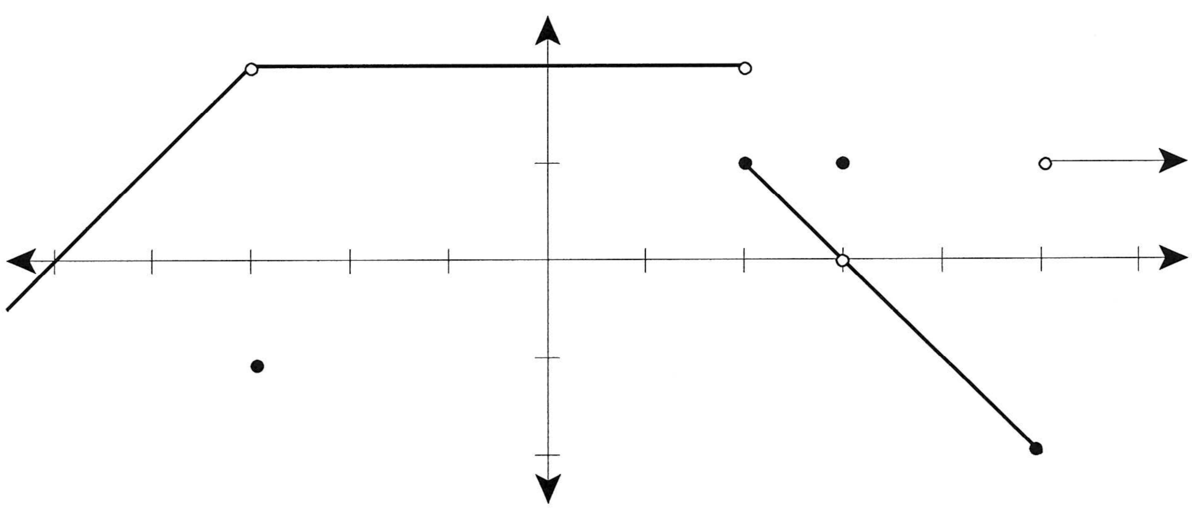
$x=-3$ $f(-3)$ & $\lim_{x\to -3}$ do not agree $x=3$ $f(3)$ and $\lim_{x\to 3}$ do not agree

$x=2$ $\lim_{x\to 2^+}$ & $\lim_{x\to 2^-}$ do not agree $x=5$ $\lim_{x\to 5^+}$ & $\lim_{x\to 5^-}$ do not agree

Excellent!

3. Find a slope function for $g(x)$.

$$s(x) = \begin{cases} 1 & \text{for } x < -3 \\ 0 & \text{for } -3 < x < 2 \\ -1 & \text{for } 2 < x < 3 \text{ and } 3 < x < 5 \\ 0 & \text{for } 5 < x \end{cases}$$



4. Suppose that $f(x) = \begin{cases} 4 & \text{for } x < 1 \\ -2x & \text{for } x = 1 \\ 10 + x & \text{for } x > 1 \end{cases}$. Evaluate the following, and make it clear how you arrived at your answers:

a) $\lim_{x \to 1^+} f(x)$

$x > 1 \Rightarrow f(x) = 10+x.$

$\Rightarrow \lim_{x \to 1^+} f(x) = \underline{\lim_{x \to 1^+}} 10+x = \underline{11}$

b) $\lim_{x \to 1^-} f(x)$

$x < 1 \Rightarrow f(x) = 4.$

$= \underline{\lim_{x \to 1^-}} 4 = \underline{4}$

c) $\lim_{x \to 1} f(x)$

Excellent!

$$\left.\begin{array}{l} \lim_{x \to 1^+} f(x) = 11 \\ \lim_{x \to 1^-} f(x) = 4 \end{array}\right] \to \underline{4 \neq 11} \Rightarrow \text{limit } \underline{\text{does not exist}}.$$

5. Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$. $= \lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x+2)}{\cancel{x-3}}$

$= \lim\limits_{x \to 3} x + 2$

$= 5$

Great!

6. Evaluate $\lim\limits_{x \to \infty} \dfrac{3x^2}{x^3 - 2x + 1}$.

$\lim\limits_{x \to \infty} \dfrac{3x^2}{x^3 - 2x + 1} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim\limits_{x \to \infty} \dfrac{\cancel{\frac{3}{x}}^{\,0}}{1 - \cancel{\frac{2}{x^2}}^{\,0} + \cancel{\frac{1}{x^3}}^{\,0}} = \dfrac{0}{1} = \boxed{0}$

Good!

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap I did good in math in high school, but now instead of formulas, they throw stuff at us where there's onl words and symbols and stuff. On our review problems for the test there was this one here, and I've got no clue how to work it out.

Help Biff by explaining as clearly as you can what the answers to these questions should be **and why**.

7. Suppose you know *f*(*x*) is continuous at 1 and that $\lim_{x \to 1^-} f(x) = 7$. Which of the following can you conclude?

(A) $\lim_{x \to 1^+} f(x) = 7$
(B) $\lim_{x \to 1} f(x) = 7$
(C) $f(1) = 7$
(D) $\lim_{x \to 1} f(x) \neq 7$
(E) None of the above *must* be true
(F) More than one of the above *must* be true

8. Suppose you know *f*(*x*) is *not* continuous at 2 and that $\lim_{x \to 2^-} f(x) = 7$. Which of the following can you conclude?

(A) $\lim_{x \to 2^+} f(x) = 7$
(B) $\lim_{x \to 2} f(x) = 7$
(C) $f(2) = 7$
(D) $\lim_{x \to 2} f(x) \neq 7$
(E) None of the above *must* be true
(F) More than one of the above *must* be true

Since #7 is continuous, $\lim_{x \to 1} f(x)$ exists and equals $f(1)$. For $\lim_{x \to 1} f(x)$ to exist, $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$ must match, so since $\lim_{x \to 1^-} f(x) = 7$, we know the others must be 7 too, so A, B, and C all must be true and the right choice is F.

Since #7 is not continuous, we can only say something didn't match, but we don't know what. It might be $\lim_{x \to 2^+} f(x) = 7$, so $\lim_{x \to 2} f(x) = 7$ too, but $f(2) \neq 7$ so it's not continuous, like what happened when $x = 3$ on the front page. It could also be that $f(2) = 7$, but $\lim_{x \to 2^+} f(x) \neq 7$, more like what happened where $x = 5$ on the front page. Bottom line is we can't be sure any one of those happens, so E is correct.

8. Evaluate $\lim_{h\to 0} \frac{\sqrt{a^2+h}-a}{h}$.

$$\lim_{h\to 0} \frac{\sqrt{a^2+h}-a}{h} \cdot \frac{\sqrt{a^2+h}+a}{\sqrt{a^2+h}+a}$$

$$\lim_{h\to 0} \frac{\cancel{a^2}+h-\cancel{a^2}}{h(\sqrt{a^2+h}+a)}$$

Beautiful.

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{a^2+h}+a)}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{a^2+h}+a} = \lim_{h\to 0} \frac{1}{\sqrt{a^2+0}+a} = \lim_{h\to 0} \frac{1}{\sqrt{a^2}+a} = \lim_{h\to 0} \frac{1}{a+a} = \boxed{\lim_{h\to 0} = \frac{1}{2a}}$$

9. Evaluate $\lim_{x\to1^+}\sqrt{\frac{x-1}{x-3}}$ and $\lim_{x\to1^-}\sqrt{\frac{x-1}{x-3}}$.

$$\lim_{x\to1^+}\sqrt{\frac{x-1}{x-3}}$$

← the bottom will produce a negative number, making the number under the sq. root negative. The $\lim_{x\to1^+}\sqrt{\frac{x-1}{x-3}}$ does not exist. Yes!

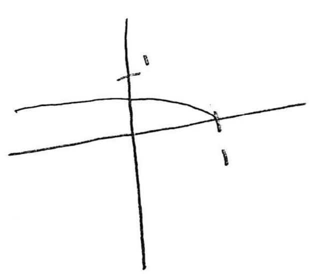
$$\lim_{x\to1^-}\sqrt{\frac{x-1}{x-3}} = \boxed{0}$$

x	f(x)
.9999	0.007071
.99999	0.0022361

Excellent!

Since both top & bottom of this fraction are negative, the number under the sq. root is positive & does exist.

also looked at graph



10. Evaluate $\lim\limits_{x\to-\infty} \dfrac{4x^3}{2x^3+\sqrt{9x^6+15x^4}}$. $\dfrac{\cdot\ \frac{1}{-x^3}}{\cdot\ \frac{1}{-x^3}} = \lim\limits_{x\to-\infty} \dfrac{-4\,\frac{x^3}{x^3}}{-2\,\frac{\cancel{x^3}}{\cancel{x^3}} + \sqrt{9\,\frac{x^6}{x^6} + 15\,\frac{x^4}{x^6}}}$

$$= \lim_{x\to-\infty} \frac{-4}{-2+\sqrt{9+\frac{15}{x^2}}}$$

$$= \frac{-4}{-2+3}$$

$$= \frac{-4}{1}$$

$$= -4$$