Exam 3 Calc 1 11/9/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int \left(\sqrt[3]{x} - \frac{1}{x} + \cos x\right) dx$.

$\int \left(x^{1/3} - 1/x + \cos x \right) dx$

$f'(x) = (3/4)x^{4/3} - \ln|x| + \sin x + C$

Check: $x^{1/3} - 1/x + \cos x$ ✓

Great!

$$\boxed{f'(x) = 3/4\, x^{4/3} - \ln|x| + \sin x + C}$$

2. Find all intervals on which $y = x^3 - x + 2$ is increasing.

Take Deriv: $y' = 3x^2 - 1$

Set $= 0$: $0 = 3x^2 - 1$

$3x^2 = 1$

$x^2 = 1/3$

$x = \pm\sqrt{1/3}$

$-.577$ $.577$

I D I

$-\sqrt{1/3}$ $\sqrt{1/3}$

$f'(-1) = 3(-1)^2 - 1$

2

Good

$f'(0) = 3(0)^2 - 1$

-1

$f'(1) = 3(1)^2 - 1$

2

$(-\infty, -\sqrt{1/3})$ Increas

$(-\sqrt{1/3}, \sqrt{1/3})$ Decrea

$(\sqrt{1/3}, \infty)$ Increas

3. Find the linear approximation $L(x)$ for $f(x) = \sqrt[3]{x}$ at 8.

$f'(x) = \frac{1}{3}x^{-2/3}$

$L(x) = f'(8)(x-8) + f(8)$

$= \frac{1}{3\times\sqrt[3]{8^2}}(x-8) + 2$

$= \frac{1}{3\times\sqrt[3]{64}}(x-8) + 2$

$L(x) = \frac{1}{12}(x-8) + 2$ //

Well done!

4. Evaluate $\lim_{x\to\infty} \frac{e^x - x - 1}{x^2}$.

$\lim_{x\to\infty} \frac{e^x - x - 1}{x^2}$ is in an indeterminate form (ie $\frac{\infty}{\infty}$)

L'H

$\Rightarrow \lim_{x\to\infty} \frac{e^x - 1}{2x}$ this is still in $\frac{\infty}{\infty}$ form

L'H

$\Rightarrow \lim_{x\to\infty} \frac{e^x}{2} = \infty$ //

Great

5. For which values of x is $f(x) = x\,e^{-x}$ concave up?

$$f'(x) = 1 \cdot e^{-x} + x \cdot -e^{-x} = e^{-x} - xe^{-x}$$

$$f''(x) = -e^{-x} - (e^{-x} - xe^{-x}) = xe^{-x} - 2e^{-x}$$

So $f''(x) = 0$ where

$$0 = xe^{-x} - 2e^{-x}$$

$$0 = e^{-x}(x-2)$$

$$x = 2$$

Now test:

$$f''(0) = 0 \cdot e^{-0} - 2e^{-0} = -2 < 0$$

$$f''(3) = 3e^{-3} - 2e^{-3} = e^{-3} > 0$$

So it's concave up for $x > 2$.

6. Jon plans to sell jet-propelled golf balls. In his trial program he sold 200 golf balls each week at a price of $100 apiece. His market research firm tells him that for each $1 he drops his price, he can sell 5 additional golf balls. What price should he charge to bring in the largest possible revenue?

Gets 5 more sales for every dollar dropped

Create equation: $f(x) = (200 + 5x)(100 - x)$

Factor: $f(x) = 20000 - 200x + 500x - 5x^2$

$-5x^2 + 20000 + 300x$

Take Deriv: $-10x + 300$

Set = 0 and factor: $10(-x + 30)$

$-x = -30$

$x = 30$

Nice Job

Jon should drop the price of his golf balls from $100 to $70 apiece which will result in the sale of 350 golf balls for a total revenue of $24,500.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "I think calculus is going to kill me! Everything is so totally confusing! I mean, I can do some of the problems, but I have no clue why anything is how it is. It seems like this whole chapter every time we take the derivative and set it equal to zero, but I have no idea why. What does it mean when you do that?"

   Explain clearly to Bunny exactly what we accomplish when we take the derivative of a function and set it equal to zero.

When we take the derivative of a function, we are learning about the slope of that function. Setting the derivative equal to zero allows us to find values of x at which the slope is zero. Typically, these places are points at which the graph of the function changes from increasing to decreasing or visa versa.

Good.

8. Let $f(x) = 12x^5 - 20x^3$ on $[-1,2]$. Find the absolute maximum and absolute minimum values of $f(x)$ on the interval.

$f(x) = 12x^5 - 20x^3$

$f'(x) = 60x^4 - 60x^2$

$0 = 60x^2(x^2 - 1)$

$60x^2 = 0$ $\quad$ $x^2 - 1 = 0$

$x^2 = 0$ $\quad$ $x^2 = 1$

$x = 0$ $\quad$ $x = \pm\sqrt{1}$

$x = \pm 1$

Excellent!

$f(-1) = 12(-1)^5 - 20(-1)^3$

$f(-1) = 8$

$f(1) = 12(1)^5 - 20(1)^3$

$f(1) = -8$ min

$f(0) = 12(0)^5 - 20(0)^3$

$f(0) = 0$

$f(2) = 12(2)^5 - 20(2)^3$

$f(2) = 224$ max

for $f(x) = 12x^5 - 20x^3$ on the interval $[-1, 2]$, the minimum of the function occurs at $x = 1$ (with a y value of -8) and the maximum occurs at $x = 2$ (with a y value of 224).

9. Show that a function of the form $f(x) = a x^2 + b x + c$, with $a \neq 0$, has an extreme value where $x = -b/2a$.

$f'(x) = 2ax + b$

$0 = 2ax + b$

$x = -\dfrac{b}{2a}$

Outstanding.

$f'(x)$ $\quad - \quad | \quad +$ $\quad$ when $a > 0$ minimum result

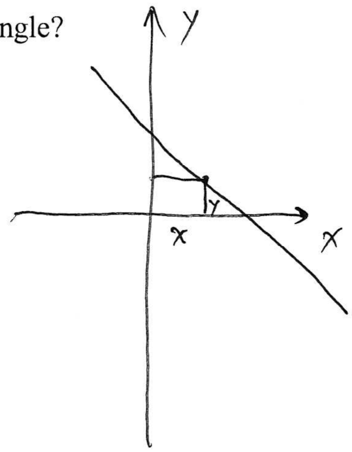
$\qquad\qquad -\dfrac{b}{2a}$

$f'(x)$ $\quad + \quad | \quad -$ $\quad$ when $a < 0$. maximum result

$\qquad\qquad -\dfrac{b}{2a}$

$\therefore f(x)$ has extreme value at $-\dfrac{b}{2a}$.

10. Jon is finishing his attic, and wants to fit in the largest rectangular storage space he can under the sloping roof around the perimeter of the room. This amounts to finding the largest (in terms of area) rectangle that fits in the first quadrant with two sides along the x- and y-axes and the upper right corner on the line $y = -\frac{2}{3}x + 6$. What are the dimensions of such a rectangle?



∴ Such rectangle should have $\frac{9}{2}$ units on x-axis and 3 units on y-axis.

$A = x \cdot y$

Nice Work!

$y = -\frac{2}{3}x + 6.$

$A = x \cdot \left(-\frac{2}{3}x + 6\right) = -\frac{2}{3}x^2 + 6x$ $\qquad y = -\frac{2}{3} \cdot \frac{9}{2} + 6 = 3.$

$\frac{dA}{dx} = -\frac{4}{3}x + 6$

$0 = -\frac{4}{3}x + 6$

$-6 = -\frac{4}{3}x \qquad x = +6 \cdot \frac{3}{4} \qquad x = \frac{9}{2}.$