**Exam 4 Calc 1 12/7/2012**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Evaluate $\int_1^3 x^2 dx$.

$$= \frac{x^3}{3}\Big|_1^3$$

$$= \frac{3^3}{3} - \frac{1^3}{3}$$ (from fundamental theorem of Calculus)

$$= 9 - \frac{1}{3}$$

Excellent!

$$= \frac{26}{3}$$

2. Find the average value of $f(x) = \sin x$ on the interval $[0, \pi]$.

$$\text{Average Value} = \frac{1}{b-a}\int_a^b f(x)\,dx =$$

$$\frac{1}{\pi - 0}\int_0^\pi \sin x\,dx =$$

$$\frac{1}{\pi} \cdot -\cos \Big|_0^\pi =$$

Excellent!

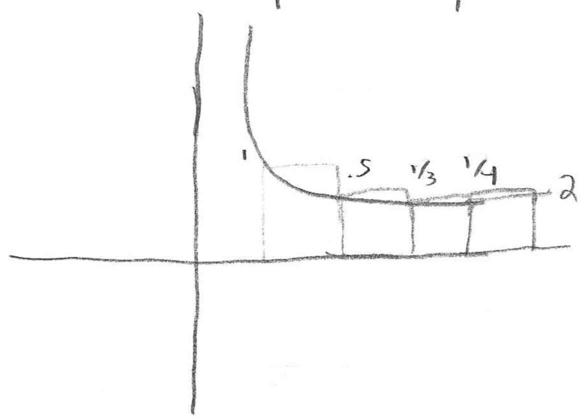
$$\frac{1}{\pi} \cdot 1 - \frac{1}{\pi} \cdot -1 =$$

$$\frac{1}{\pi} + \frac{1}{\pi} =$$

$$\boxed{\frac{2}{\pi}}$$

3. If you use a left-hand sum with $n = 4$ subdivisions to approximate $\int_1^5 \frac{1}{x}\,dx$, what are:

$\Delta x = 1$ $\qquad \frac{5-1}{4} \quad \frac{4}{4}$




$\bar{x}_1 = 1$

$\bar{x}_2 = 2$

$\bar{x}_3 = 3$

$\bar{x}_4 = 4$

$f(\bar{x}_1) = 1$

$f(\bar{x}_2) = \frac{1}{2}$

Great

$f(\bar{x}_3) = \frac{1}{3}$

$f(\bar{x}_4) = \frac{1}{4}$

$$\sum_{k=1}^{4} f(\bar{x}_k)\cdot\Delta x = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)\cdot 1 = \frac{25}{12}$$

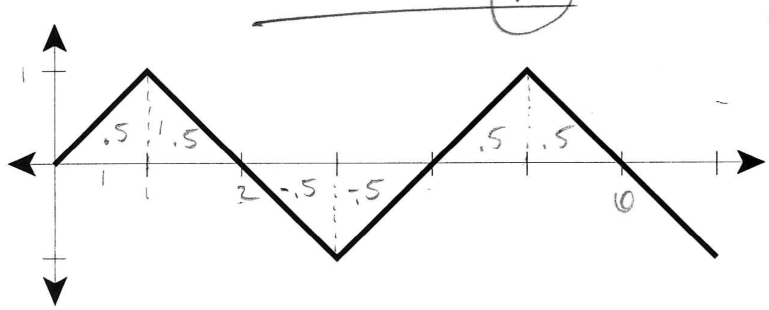
$\frac{12}{12} \quad \frac{6}{12} \quad \frac{4}{12} \quad \frac{3}{12}$

4. For the function $f(x)$ whose graph is shown below, find

a) $\int_0^2 f(x)\,dx$ $\quad \frac{1}{2}bh + \frac{1}{2}bh = \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1) = 1$

b) $\int_0^6 f(x)\,dx$ $\quad 1 - 1 + 1 = 1$

Excellent!



5. Evaluate $\int \frac{x}{\sqrt{9-x^2}} \cdot dx$

let $u = 9 - x^2$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$+C$

$= \int \frac{x}{\sqrt{u}} \, dx$

$= \int x u^{-1/2} \frac{du}{-2\cancel{x}}$

$= -\frac{1}{2} \int u^{-1/2} \, du$

$= -\frac{1}{\cancel{2}} \cancel{2} u^{\frac{1}{2}}$

$= -u^{\frac{1}{2}} + C$

$= -(9-x^2)^{\frac{1}{2}} + C$

Well done!

6. Suppose $\int_0^2 f(x)\,dx = 5$, $\int_2^5 f(x)\,dx = -1$, $\int_0^2 g(x)\,dx = 7$, and $\int_2^5 g(x)\,dx = 2$.

a) Evaluate $\int_0^2 3f(x)\,dx$.

$= 3\int_0^2 f(x)\,dx = 3 \cdot 5 = 15$

b) Evaluate $\int_0^5 f(x)\,dx$.

$= \int_2^5 f(x)\,dx + \int_0^2 f(x)\,dx = -1+5 = 4$

Excellent!

c) Evaluate $\int_2^5 [f(x) - g(x)]\,dx$.

$= \int_2^5 f(x)\,dx - \int_2^5 g(x)\,dx$

$= -1 - 2 = -3$

7. a) Evaluate $\frac{d}{dx}\int_0^x \frac{1}{1+t^3}\,dt$.

$= \frac{1}{1+x^3}$ ( from Fundamental theorem g calculus)

b) Evaluate $\frac{d}{dx}\int_0^{5x} \frac{1}{1+t^3}\,dt$.

$= 5\left(\frac{1}{1+(5x)^3}\right)$

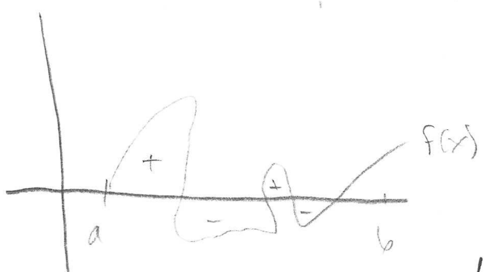
[from Chain rule & FTC]

Great

8. The first edition of the Cliff'sNotes Calculus Quick Review says on page 116 that the area $A$ of the region bounded by the graph of $f(x)$, the $x$ axis, and the lines $x = a$ and $x = b$ is given by $\left|\int_a^b f(x)dx\right|$ when the function $f(x)$ is sometimes above and sometimes below the $x$-axis.

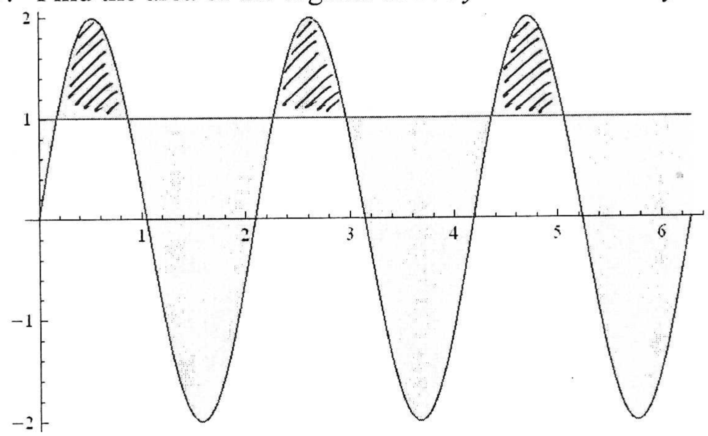
Explain in at least a few sentences why the absolute value bars in this expression do or don't accurately express the actual area bounded by these curves.

This does not accurately express the area because it is doing the absolute value after the integral has been taken. If you like bad in class metaphors, it's like the cops showing up after the gun fight. :) When finding the area it doesn't matter whether the bounded region is below or above the x axis, it's just painting in the attic or the basement, still painti[ng]. The problem with the given equation is that it allows the positive values to be cancelled out by the negative values; ~~then~~, later takes the absolute value after the fact, rather than doing that to the (−) values in the first place, thus understating area.




Wonderful!

9. Find the area of the regions above $y = 1$ but below $y = 2\sin(3x)$ between $x = 0$ and $x = 2\pi$.




Intersection: $1 = 2\sin(3x)$

$\frac{1}{2} = \sin(3x)$

$3x = \arcsin 1/2$

$3x = \frac{\pi}{6}$ or $3x = \frac{5\pi}{6}$

$x = \frac{\pi}{18}$ or $x = \frac{5\pi}{18}$

So the area of one bump is:

$$\int_{\pi/18}^{5\pi/18} [2\sin(3x) - 1]\,dx$$

Let $u = 3x$

$\frac{du}{dx} = 3$

$\frac{du}{3} = dx$

$$= \int_{\pi/18}^{5\pi/18} [2\sin u - 1] \cdot \frac{du}{3}$$

$$= \frac{1}{3} \cdot \Big[-2\cos u - u\Big]_{x=\pi/18}^{x=5\pi/18}$$

$$= \Big[\frac{-2}{3}\cos 3x - \frac{1}{3}\cdot 3x\Big]_{\pi/18}^{5\pi/18}$$

$$= \left(-\frac{2}{3}\cos\frac{5\pi}{6} - \frac{5\pi}{18}\right) - \left(\frac{-2}{3}\cos\frac{\pi}{6} - \frac{\pi}{18}\right)$$

$$= -\frac{2}{3}\cdot\frac{-\sqrt{3}}{2} - \frac{5\pi}{18} + \frac{2}{3}\frac{\sqrt{3}}{2} + \frac{\pi}{18}$$

$$= \frac{\sqrt{3}}{3} - \frac{4\pi}{18} + \frac{\sqrt{3}}{3}$$

$$= \frac{2\sqrt{3}}{3} - \frac{2\pi}{9}$$

So three bumps like that have

Area $= 2\sqrt{3} - \frac{2\pi}{3}$

10. Evaluate $\int_0^5 (x+1)\sqrt{25-x^2}\,dx$. $= \int_0^5 x\sqrt{25-x^2}\,dx + \underbrace{\int_0^5 \sqrt{25-x^2}\,dx}$

$= \frac{1}{4}$ circle with $r=5$

$= \frac{1}{4}\cdot\pi\cdot 5^2$

$= \frac{25\pi}{4}$

Let $u = 25-x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$

So $= \int_{x=0}^{x=5} x\cdot u^{1/2}\cdot\frac{du}{-2x}$

$= \frac{-1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_{x=0}^{x=5}$

$= -\frac{1}{3}(25-x^2)^{3/2}\Big|_0^5$

$= -\frac{1}{3}\cdot 0 - \frac{-1}{3}\cdot 25^{3/2}$

$= \frac{125}{3}$

So the total integral has value $\frac{125}{3} + \frac{25\pi}{4}$