

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. a) Find the linear approximation  $L(x)$  for  $f(x) = \sqrt[3]{x}$  at 8.

$$\begin{aligned} f(x) &= x^{1/3} \\ f'(x) &= \frac{1}{3} x^{-2/3} \\ f'(8) &= \frac{1}{3(8^{2/3})} = \frac{1}{12} \end{aligned}$$

$$f(8) = \sqrt[3]{8} = 2$$

$$\boxed{L(x) = \frac{1}{12}(x-8) + 2}$$

- b) Use your linearization from part a to approximate  $\sqrt[3]{8.01}$ .

$$L(8.01) = \frac{1}{12}(8.01-8) + 2$$

$$\boxed{L(8.01) = 2.00083}$$

- c) Use your linearization from part a to approximate  $\sqrt[3]{8.1}$ .

$$L(8.1) = \frac{1}{12}(8.1-8) + 2$$

$$\boxed{L(8.1) = 2.0083}$$

- d) Use your Linearization from part a to approximate  $\sqrt[3]{27}$ .

$$L(27) = \frac{1}{12}(27-8) + 2$$

$$\boxed{L(27) = 3.583}$$

2. a) Find the linear approximation  $L(x)$  for  $f(x) = \arctan x$  at 0.

$$f'(x) = \frac{1}{1+x^2} \quad f(0) = \tan^{-1}(0) = 0$$

$$f'(0) = \frac{1}{1+0^2} = 1 \quad L(x) = 1(x-0) + 0$$

$$\boxed{L(x) = x}$$

- b) Use your Linearization from part a to approximate  $\arctan 0.1$ .

$$\boxed{L(0.1) = 0.1}$$

Wonderful!

Excellent!