Exam 1 Calc 3 9/28/2012

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to x.

2. Show that $\lim_{(x,y)\to(0,0)} \frac{x-y}{x^2+y^2}$ does not exist.

3. Suppose that w is a function of x, y, and z, each of which is a function of s and t. Write the Chain Rule formula for $\dfrac{\partial w}{\partial t}$. Make very clear which derivatives are partials.

4. Let $f(x, y) = \sqrt{x + xy^2}$. Find the directional derivative of f at the point (5,2) in the direction of the vector $\langle 1,1 \rangle$.

5. Find an equation for the plane tangent to the surface $z = \ln(1 + xy)$ at the point $(2, 3, \ln 7)$.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this Calc 3 stuff is killing me. It's like there's not just three variables, there's three ways to think about everything too, and I'm lucky if I can figure out one. Our T.A. did one of those max problems, and instead of telling if it was a max or a min at this one point by doing the sign on f_{xx}, he did the sign on f_{yy}, but the book didn't say that. He said it was easier, but I don't get it. I mean, what if f_{xx} was the other sign than f_{yy}? Is it both a max and min then?

   Explain clearly to Biff whether it was okay for his instructor to use f_{yy}, and what to expect of the signs on f_{xx} and f_{yy} at an extremum.

8. Find and classify the critical points of $V(x, y) = xy(18 - x - y)$.

9. At which points on the surface $z = x^2 - y^2$ is there at least one direction in which the directional derivative is at least 1?

10. Find the directions in which the function $f(x, y) = 4x^2 - y^2$ has zero change at the point (a, b).

Extra Credit (5 points possible):
Let $f(x, y) = (x^2 + y^2)^{2/3}$. Show that

$$f_x(x,y) = \begin{cases} \dfrac{4x}{3\left(x^2+y^2\right)^{1/3}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$$