Quiz 2 Calculus 3 10/5/2012

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$.

Along $x=0$ $\rightarrow$ $\lim\limits_{(0,y)\to(0,0)} \dfrac{0.y}{0^2+y^2} = \lim\limits_{(0,y)\to(0,0)} \dfrac{0}{y^2} = 0$

Along $x=y$ $\rightarrow$ $\lim\limits_{(x,x)\to(0,0)} \dfrac{x.x}{x^2+x^2} = \lim\limits_{(x,x)\to(0,0)} \dfrac{x^2}{2x^2} = \dfrac{1}{2}$

Since the limit approaches different limits along different lines, the limit doesn't exist

Great

2. Find an equation for the plan tangent to $z = \dfrac{1}{3+xy^2}$ at the point (3,–2,1/15).

$z = \dfrac{1}{3+xy^2} = (3+xy^2)^{-1}$

$z_x = -1(3+xy^2)'(3+xy^2)^{-1-1}$

$z_x = -1(y^2)(3+xy^2)^{-2} = \dfrac{-y^2}{(3+xy^2)^2}$

$z_y = -1(3+xy^2)'(3+xy^2)^{-1-1}$

$= -1(2xy)(3+xy^2)^{-2} = \dfrac{-2xy}{(3+xy^2)^2}$

$z_x(3,-2) = \dfrac{-(-2)^2}{(3+3.(-2)^2)^2} = \dfrac{-4}{225}$, $z_y(3,-2) = \dfrac{12}{225}$

the equation for the plane tangent to z:

$z - \dfrac{1}{15} = \dfrac{-4}{225}(x-3) + \dfrac{12}{225}(y+2)$

Wonderful.

3. Find and classify all critical points of $V(x, y) = xy(24 - x - y)$. $= 24xy - x^2y - xy^2$

$$\begin{cases} V_x(x,y) = 24y - 2xy - y^2 = 0 \\ V_y(x,y) = 24x - x^2 - 2xy = 0 \end{cases} \Rightarrow x^2 - 24x + 24y - y^2 = 0 \Leftrightarrow (x-y)(x+y-24) = 0 \Leftrightarrow \left[\begin{array}{l} x = y \\ x = 24 - y \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} 24y - 2y^2 - y^2 = 0 \\ 24y - 2y(24-y) - y^2 = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} y(24-3y) = 0 \\ y(y-24) = 0 \end{array}\right.$$

Excellent!

$$\Leftrightarrow \left[\begin{array}{lll} y = 0 = x & D(0,0) = 0 \times 0 - 24^2 < 0 & \Rightarrow \text{saddle point} - (0,0) \\ y = 8 = x & D(8,8) = 128 - (24 - 2.8 - 2.8)^2 > 0 \quad f(x,x) < 0 & \Rightarrow (8,8) \text{ local maximum point} \\ y = 0, x = 24 & D(24,0) = 0 - (24 - 2.24)^2 < 0 & \Rightarrow (24,0) - \text{saddle point} \\ y = 24, x = 0 & D(0,24) = 0 - (24 - 2.24)^2 < 0 & \Rightarrow (0,24) - \text{saddle point} \end{array}\right.$$

$V_{xx} = -2y$, $V_{yy} = -2x$

$V_{xy} = 24 - 2x - 2y$

4. Find and classify all critical points of $f(x, y) = e^y(y^2 - x^2)$.

$$\begin{cases} f_x(x,y) = -2e^y x = 0 \\ f_y(x,y) = e^y 2y + e^y y^2 - x^2 e^y = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ e^y 2y + e^y y^2 = 0 \end{cases} \Leftrightarrow \left[\begin{array}{l} x = 0, \ y = 0 \\ x = 0, \ y = -2 \end{array}\right.$$

$f_{xx}(x,y) = -2e^y$ | $f_{xy}(x,y) = -2xe^y$

$f_{yy}(x,y) = 2(e^y + e^y y) + e^y 2y + e^y y^2 - x^2 e^y$

$\Rightarrow f_{xx}(0,0) = -2$; $f_{yy}(0,0) = 2(1+0) + 0 + 0 - 0 = 2$; $f_{xy}(0,0) = 0$

$f_{xx}(0,-2) = -2e^{-2}$; $f_{yy}(0,-2) = 2(e^{-2} - 2e^{-2}) + e^{-2} \times (-4) + e^{-2} \times 4 = -2e^{-2}$; $f_{xy}(0,-2) = 0$

$D(0,0) = -2 \times 2 - 0 < 0 \Rightarrow (0,0) - \text{saddle point}$

Well done!

$D(0,-2) = -2e^{-2} \times (-2e^{-2}) - 0 = 4e^{-4} > 0 \quad f_{xx}(0,-2) < 0$

$\Rightarrow (0,-2) - \text{local maximum point}$