Exam 2   Real Analysis 1   11/9/2012

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the global definition on continuity.

2. State Fermat's Theorem.

3. a) State the definition of a compact set.

   b) State the Heine-Borel Theorem.

4. a) State the definition of the derivative of f at a.

   b) Give an example of a function defined on $\mathbb{R}$ that is not differentiable anywhere.

5. State and prove the Product Rule for Derivatives.

6. State and prove the Mean Value Theorem.

7. Show that if a function $f\colon D \to \mathbb{R}$ is differentiable at some $a \in D$, then f is also continuous at a.

8. State and prove Rolle’s Theorem.

9. Show directly from the definitions that any subset of the reals with a finite number of elements is closed.

10. Suppose that f: $[a, b] \to \mathbb{Q}$ is continuous on $[a, b]$. Prove that f is constant on $[a, b]$.