Problem Set 4 Real Analysis Due 9/23/2012

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Prove or give a counterexample: If $\{a_n\}$ diverges and $\{b_n\}$ diverges, then it must be the case that $\{a_n \cdot b_n\}$ diverges also.

2. Prove that statements (a) and (b) of Remark 2.4.3 are equivalent.

3. Give an example of a sequence that diverges to $-\infty$ but is not eventually decreasing.

4. Let $a_{n+1} = 1 + \frac{a_n}{2}$, and $a_1 = 1$. Show that this sequence converges.

5. Show that a finite set has no accumulation points.

6. Give an example of a sequence where $\lim_{n\to\infty} |a_{n+1} - a_n| = 0$, but the sequence diverges.

7. Prove that every Cauchy sequence is bounded *without* using the fact that in $\mathbb{R}$ Cauchy sequences converge.

8. Prove that if a_n and b_n are Cauchy sequences, then so is $\{a_n - b_n\}$.

9. Do Exercise 2.5.10.