Problem Set 5   Real Analysis   Due 9/30/2012

Several of these problems will be graded, with each graded problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Do #3 in §2.7.

2. Do #4 in §2.7.

3. Suppose that $\lim_{x \to \infty} f(x) = A$ and $\lim_{x \to \infty} g(x) = B$. Prove (directly from the definition) that $\lim_{x \to \infty} [f(x) + g(x)] = A + B$.

4. Suppose that $\lim_{x \to \infty} f(x) = A$ and $\lim_{x \to \infty} g(x) = B$. Prove (directly from the definition) that $\lim_{x \to \infty} [f(x) \cdot g(x)] = A \cdot B$.

5. Suppose that $\lim_{x \to \infty} f(x) = A$ and $\lim_{x \to \infty} g(x) = B$. Prove (directly from the definition) that if $f(x) \leq g(x)$ for each $x \in D$, then $A \leq B$, or give a counterexample.

6. Suppose that $\lim_{x \to \infty} f(x) = A$ and $\lim_{x \to \infty} g(x) = B$. Prove (directly from the definition) that if $f(x) < g(x)$ for each $x \in D$, then $A < B$, or give a counterexample.