Exam 1b Calc 3 9/27/2013

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function f(x, y) with respect to x.

2. Find an equation for the plane tangent to $z = 9 - 2x^2 - y^2$ at the point (–1,2,3).

3. Show that $\lim_{(x,y)\to(0,0)} \frac{(x-y)^2}{x^2+y^2}$ does not exist.

4. Suppose that f is a function of x and y, each of which is a function of t, u, and v. Write the Chain Rule formula for $\frac{\partial f}{\partial t}$. Make very clear which derivatives are partials.

5. Let $f(x,y)=\sqrt{4-x+y^2}$. Find the directional derivative of f at the point (–4,1) in the direction of the vector $\langle 1,2\rangle$.

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{b}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this Calc 3 stuff is killing me. There's all this stuff where you can't just solve an equation and have an answer, you know? It's totally unfair. Like, there was this question they asked us when we reviewed in class, about like, if there was this circle around the origin where everything was a max, so it's like the lip of a volcano, you know? Then the question was does there have to be a local min somewhere inside that circle. There's not even a freaking formula, so how should I know?"

   Explain clearly to Biff whether the conditions he describes are sufficient to draw a conclusion, and why.

8. Find the maximum value of $f(x, y) = xy - y + 3$ subject to the constraint $2x + 3y = 6$.

9. Describe the collection of points on the surface $z = 3 - xy$ where the slope in the direction of greatest increase is equal to 1.

10. Jon is planning to build a large sculpture in the math lounge. It will consist of two paraboloids, one with equation $z = -1 - x^2 - y^2$ and the other with equation $z = 1 + x^2 + y^2$. There will also be a plane tangent to both of these paraboloids. Which points on the paraboloids can work as points of tangency for this plane?

Extra Credit (5 points possible):

a) What is the maximum value of the function $f(x,y)=\dfrac{1}{x^2+y^2+1}$?

b) What is the maximum directional derivative of the function $f(x,y)=\dfrac{1}{x^2+y^2+1}$?