Exam 3b Calculus 3 11/26/2013

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Give a parametrization and bounds for t for the bottom half of the circle with radius 3, centered at the origin and traversed counterclockwise.

2. If $\mathbf{F}(x,y,x) = 3xy\ \mathbf{i} + 6yz\ \mathbf{j} + xz\ \mathbf{k}$, evaluate curl $\mathbf{F}$.

3. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = xy\,\mathbf{i} + 2y\,\mathbf{j}$ and C is the line segment from (0,0) to (3,4).

4. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = 15x^4y\,\mathbf{i} + 3x^5\,\mathbf{j}$ and C is the line segment from (1,3) to (2,5).

5. Let $\mathbf{F}(x,y) = 3xy\,\mathbf{i} + x^2\,\mathbf{j}$ and C be the line segment from (0,0) to (3,0), followed by the line segment from (3,0) to (0,3), then the line segment from (0,3) to the origin. Set up an integral (or integrals) involving only scalar quantities for $\int_C \mathbf{F} \cdot d\mathbf{r}$.

6. Show that for any vector field in $\mathbb{R}^3$ whose component functions have continuous second-order partial derivatives, div curl $\mathbf{F} = 0$. Make it clear why the requirement about continuity is important.

7. Biff is a Calc 3 student at Enormous State University and he's having some trouble. Biff says "Dude, this stuff is killing me. I'm pretty good with it when you're dropping things into a formula, but now there's all these theorems and stuff. Our TA keeps saying things like how circulation is always zero in a conservative vector field, and I don't know what any of that means. Can you help me?"

   Explain as clearly as possible to Biff what each of those terms means, and why his TA's conclusion is valid.

8. Suppose that $\mathbf{F}$ is a vector field whose divergence is zero everywhere and S is a tetrahedron consisting of the four sides S_1, S_2, S_3, and S_4. If you know that $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = 2$, $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S} = 2$, and $\iint_{S_3} \mathbf{F} \cdot d\mathbf{S} = 2$, then what can be said about $\iint_{S_4} \mathbf{F} \cdot d\mathbf{S}$? Explain your reasoning.

9. Suppose $\mathbf{F}(x, y, z) = P(x, y, z)\,\mathbf{i} + R(x, y, z)\,\mathbf{k}$, and let S be the rectangular portion of the plane $y = 2$ with vertices (0,2,0), (3,2,0), (3,2,5), and (0,2,5), oriented in the direction of the positive y-axis. Show why $\iint_S \mathbf{F} \cdot d\mathbf{S}$ must equal zero.

10. Let $\mathbf{F}(x,y,z) = \langle y - z, z - x, x - y \rangle$. Let S be the portion of the sphere $x^2 + y^2 + z^2 = 25$ below $z = 4$, with outward orientation. Evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

Extra Credit (5 points possible):

Compute $\int_C \left\langle \frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2} \right\rangle \cdot d\mathbf{r}$ for C the counterclockwise circle centered at the origin of radius R.