Exam 1a Real Analysis 1 10/3/2014

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of the limit of a function $f(x)$ as x approaches $+\infty$.

2. a) State the definition of an accumulation point of a set S.

   b) State the definition of a Cauchy sequence.

3. a) Give an example of a sequence that converges to 2.

   b) Give an example of a function that diverges to $+\infty$, but that is not eventually increasing.

4. a) Give an example of two sequences that diverge, but whose sum converges.

   b) Give an example of two sequences that diverge, but whose product converges.

5. a) State the Triangle Inequality.

   b) State the Monotone Convergence Theorem.

6. Show that any convergent sequence is bounded.

7. Show that if a function f has a limit as x approaches a, then that limit is unique.

8. State and prove the Bolzano-Weierstrass Theorem for Sets.

9. Show that any finite set has no accumulation points.

10. A theorem states "Suppose that $\lim_{x \to \infty} f(x) = A$ and $\lim_{x \to \infty} g(x) = B$, where f and g are functions with domain D. Then $\lim_{x \to \infty}[f(x) \cdot g(x)] = A \cdot B$." Why is the requirement that f and g have the same domain important?