Exam 2 Real Analysis 1 11/7/2014

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the local definition of continuity.

2. a) State the definition of a relative maximum.

   b) State Fermat's Theorem

3. a) Give an example of a function f that is differentiable at $x = a$ such that $f'(a)$ exists, with $f'(a) \neq 0$, but yet f attains a relative extremum at $x = a$.

   b) Give an example of a function f that is continuous at $x = a$, not differentiable at $x = a$, but yet f attains a relative extremum at $x = a$.

4. a) State the definition of a compact set.

   b) State the Heine-Borel Theorem.

   c) Give an example of a set with an open cover that has no finite subcover.

5. State and prove the Difference Rule for Derivatives.

6. Show that if a function $f{:}D \to \mathbb{R}$ is differentiable at some $a \in D$, then f is also continuous at a.

7. Let f:[a, b] → ℝ be continuous, and $\{x_n\}$ be a sequence in [a, b] converging to c. Show $\lim_{n\to\infty} f(x_n) = f\left(\lim_{n\to\infty} x_n\right)$.

8. State and prove the Mean Value Theorem.

9. State and prove (Bolzano’s) Intermediate Value Theorem.

10. Suppose that f is a differentiable function from $\mathbb{R}$ to $\mathbb{R}$, and that g is a function from $\mathbb{R}$ to $\mathbb{R}$ for which $f \cdot g$ is differentiable on $\mathbb{R}$. What can you say about the differentiability of g?