Exam 1a　　Calc 1　　9/25/2015

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 and 2:

1. Find the following limits:
   a) $\lim_{x\to 3^-} g(x) = \underline{0}$
   b) $\lim_{x\to 3^+} g(x) = \underline{1}$
   c) $\lim_{x\to 3} g(x) = \underline{DNE}$, Because the $\lim_{x\to 3^-} g(x) \neq \lim_{x\to 3^+} g(x)$
   d) $\lim_{x\to 5^+} g(x) = \underline{1}$
   e) $\lim_{x\to 5} g(x) = \underline{1}$

Great

2. For which values of x does the function fail to be continuous?

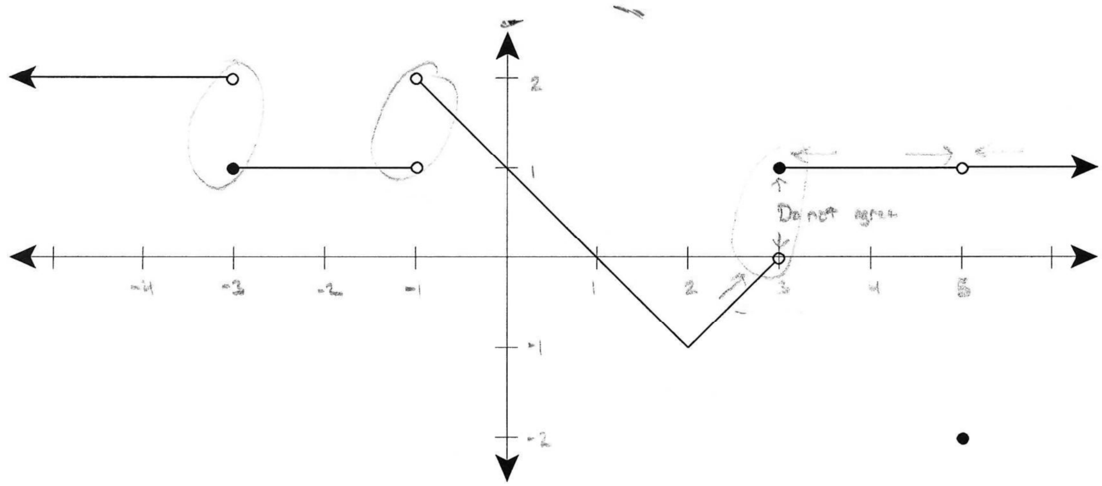
$\underline{x=-3}$ $\lim_{x\to -3^-}$ and $\lim_{x\to -3^+}$ do not agree

$\underline{x=-1}$ $\lim_{x\to -1^-}$ and $\lim_{x\to -1^+}$ do not agree

$\underline{x=3}$ $\lim_{x\to 3^-}$ and $\lim_{x\to 3^+}$ do not agree

$\underline{x=5}$ $\lim_{x\to 5^-}$ and $\lim_{x\to 5^+}$ do not agree with $g(5)$ which is discontinuous

Excellent!



3. Fill in the table of values and guess the value of the limit $\lim_{y \to 2} \frac{y^2 - y - 2}{y^2 + y - 6}$.

y	$f(y)$	y	$f(y)$
2.002	.60016	1.998	.59984
2.001	.60008	1.999	.59992
2.0001	.600008	1.9999	.599992

$$\lim_{y \to 2} \frac{y^2 - y - 2}{y^2 + y - 6} = .6$$ Good

4. Evaluate $\lim_{t \to 2.5} \frac{-16t^2 + 100}{t - 2.5}$.

$$= \lim_{t \to 2.5} \frac{\cancel{(t-2.5)}(-16t - 40)}{\cancel{(t-2.5)}}$$

$$(-16t^2 + 100)$$

$$(t - 2.5)(-16t - 40)$$

$$-16t^2 - 40t + 40t + 100$$

$$= \lim_{t \to 2.5} \frac{(-16t - 40)}{1}$$

$$= \frac{-16(2.5) - 40}{1}$$

$$= \frac{-80}{1}$$

$$= -80$$

Great.

5. Evaluate $\lim_{x\to 2}\sqrt{\dfrac{4+2x^3}{3x-1}}$, **carefully identifying which limit law you use at each step.**

$$\lim_{x\to 2}\sqrt{\frac{4+2x^3}{3x-1}} = \sqrt{\frac{\lim_{x\to 2} 4+2x^3}{\lim_{x\to 2} 3x-1}}$$ * Roots Law * Hm. *q. Law*

$$= \sqrt{\frac{\lim_{x\to 2} 4+\lim_{x\to 2} 2x^3}{\lim_{x\to 2} 3x - \lim_{x\to 2} 1}}$$ * Sum/Difference law *

$$= \sqrt{\frac{\lim_{x\to 2} 4+2(\lim_{x\to 2} x^3)}{3(\lim_{x\to 2} x) - \lim_{x\to 2} 1}}$$ * Constant Multiple law *

$$= \sqrt{\frac{\lim_{x\to 2} 4+2(\lim_{x\to 2} x)^3}{3(\lim_{x\to 2} x) - \lim_{x\to 2} 1}}$$ * power law *

$$= \sqrt{\frac{4+2(2)^3}{3(2)-1}}$$ * Constant & Law x * Law

$$= \sqrt{\frac{4+16}{6-1}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$$

Nice!

6. Evaluate $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{\theta}$ by any means you prefer. **Provide good justification for your conclusion.**

$f(\theta)$ $$\boxed{\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = 0}$$

numerically Z.

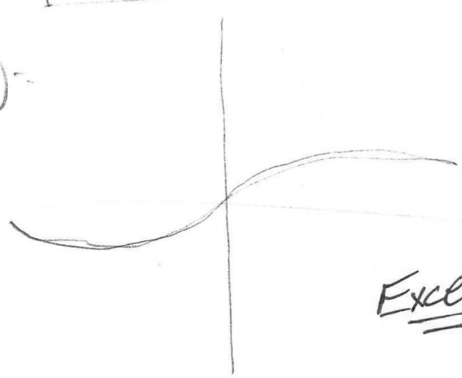
θ	$f(\theta)$
.1	.04996
.01	.005
.003	.0015

closer to zero

θ	$f(\theta)$
-.1	-.05
-.01	-.005
-.003	-.0015

closer to zero

$f(\theta)$=



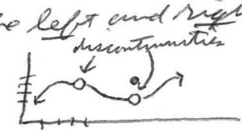
Excellent!

graphically supports

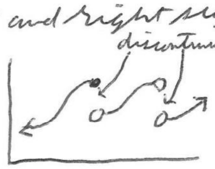
7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod. I always thought math was okay because I could learn to get the answers to problems, but now it's all different! There's, like, these, like, questions where we're supposed to *classify* stuff instead of work out an answer. Ohmygod, is this biology or something? So, like, we're supposed to be able to tell when a discontinuity is removable or a jumpy one or infinite, and I have no clue how you do that. Ohmygod!"

   Help Bunny by explaining as clearly as you can what the difference is between these three kinds of discontinuities.

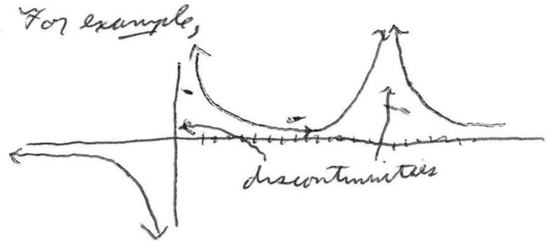
A removable discontinuity occurs when a function is ~~un~~defined at a particular point differently than ~~at a particular point, yet~~ the limits to the left and right sidedness of this point which are identical. For example,



A jump discontinuity is defined as a function, at a particular point, having the left and right sidedness of the limit at that point differ. For example,



An infinite discontinuity occurs when the slope of the graph as it approaches a particular point approaches infinity. This can be either negative or positive infinity. For example,



Excellent!

8.

9. Find the value for the constant c that makes the function $f(x)=\begin{cases} x^2-c & \text{for } x<5 \\ 3x+2c & \text{for } x\geq 5 \end{cases}$ continuous.

If it's continuous, the limits from left and right have to agree when $x=5$, so

$$(5)^2-c = 3(5)+2c$$

$$25-c = 15+2c$$

$$10 = 3c$$

$$c = \frac{10}{3}$$

10. Evaluate $\lim_{h\to 0}\dfrac{\dfrac{1}{(h+a)^2}-\dfrac{1}{a^2}}{h}$.

$$= \lim_{h\to 0} \frac{\dfrac{a^2-(h+a)^2}{(h+a)^2\cdot a^2}}{h}$$ Common Denominator

$$= \lim_{h\to 0} \frac{a^2-(h^2+2ah+a^2)}{a^2(h+a)^2}\cdot\frac{1}{h}$$ Distribute

$$= \lim_{h\to 0} -\frac{h^2+2ah}{ha^2(h+a)^2}$$ Simplify

$$= \lim_{h\to 0} -\frac{\cancel{h}(h+2a)}{\cancel{h}a^2(h+a)^2}$$ Factor

$$= \lim_{h\to 0} -\frac{\overset{0}{h}+2a}{a^2(\underset{0}{h}+a)^2}$$ Take limit

$$= -\frac{2a}{a^4}$$ Simplify

$$= -\frac{2}{a^3}$$