Exam 3 Calc 1 11/13/2015

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find all critical points of $f(x) = x^3 - 12x$.

$F'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

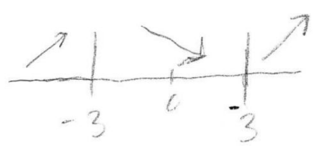
$0 = 3(x-2)(x+2)$

critical points $= \pm 2$

Great

2. Determine where $f(x) = x^3 - 12x$ has local maxima and minima.

intervals $\rightarrow$ $(-\infty, \overset{-3}{-2}]$, $[-2, \overset{0}{2}]$, $[\overset{3}{2}, \infty)$



$(-\infty, -2] =$ increasing

$[-2, 2] =$ decreasing

$[2, \infty) =$ increasing

$f'(-3) = 3(-3)^2 - 12 = 15$

$f'(0) = 3(0)^2 - 12 = -12$

$f'(3) = 3(3)^2 - 12 = 15$

min $= 2$
max $= -2$

Good!

3. Find the linear approximation $L(x)$ for $f(x) = \sqrt[4]{x}$ at 16.

$f(x) = \sqrt[4]{x}$

$L(x) = f(a) + f'(a)(x-a)$

$\sqrt[4]{16} = 2$

$L(x) = 2 + \left(\frac{1}{32}(x-16)\right)$

$f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$

$f'(16) = \frac{1}{32}$

$L(x) = 2 + \frac{1}{32}x - \frac{1}{2}$

$\boxed{L(x) = \frac{3}{2} + \frac{1}{32}x}$

Great

Lets say $x = 16.02$

$L(16.02) = \frac{3}{2} + \frac{1}{32}(16.02)$

$L(16.02) = 2.000625$

I like it.

4. Find two positive real numbers such that they add to 40 and their product is as large as possible.

$x + y = 40 \Rightarrow y = 40 - x$

$xy = \text{big}$ substitute

$x(40-x) = f(x)$

$40x - x^2 = f(x)$

$f'(x) = 40 - 2x$

$0 = 40 - 2x$

$\frac{2x}{2} = \frac{40}{2}$

$x = 20$

Great Job

$20 + y = 40$

$\boxed{\begin{matrix} y = 20 \\ x = 20 \end{matrix}}$

5. Evaluate $\lim_{x\to\infty} \frac{\ln x}{x}$.

$$\lim_{x\to\infty} \frac{\ln x}{x}$$

is it a fraction? Yes!

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{1/x}{1}$$

Is it a race to the finish? yes!

$$= \lim_{x\to\infty} 1/x$$

$$= \underline{0}$$

Dividing 1 by a humongous number will be very, very, close to zero

Excellent!

6. Apply Newton's Method to $f(x) = x^3 - 5$ with initial guess $x_0 = 2$ to calculate x_1.

$x_0 = 2$

$f(2) = (2)^3 - 5 = 3$

$f'(x) = 3x^2 - 0$

$f'(2) = 3(2)^2 = 12$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 2 - \frac{f(2)}{f'(2)}$

$x_1 = 2 - \frac{3}{12}$

$x_1 = \frac{21}{12} = \boxed{\frac{7}{4}}$

Great

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "I think calculus is the hardest thing ever! Just when you think it's getting easy, like with the Loopy-tall rule thingy? At first I thought it was really easy. But then there was this one on the test, like a limit for $x^2 - 2x$ over $3x - 2$, like if x is 0, and I did the Loopy-tall thingy and they gave me no credit, not even partial. That's so unfair!"

   Explain clearly to Bunny what she should understand about using L'Hôpital's Rule on her function..

Well bunny, in order to use L'Hôpitals Rule correctly you must first determine if the function is in the indeteminate form, that is $\frac{0}{0}$, $\frac{\infty}{\infty}$ or $\frac{-\infty}{-\infty}$ etc when it is in this form it would be correct to use the derivatives b/c you're starting off with an "equal fraction", the slopes would be zero for both so you can use l'hopitals In the function $\lim_{x \to 0} \frac{x^2 - 2x}{3x-2}$ if you plug in 0 you get $-\frac{0}{2}$ which is not applicable for l'hopitals rule. b/c its not indeterminate.

Excellent!

8. The manager of a large apartment complex knows from experience that 90 units will be occupied if the rent is 344 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 4 dollar increase in rent. Similarly, one additional unit will be occupied for each 4 dollar decrease in rent. What rent should the manager charge to maximize revenue?

Revenue = Units · \$ per Unit

344 \$ per month

90 units

	90	$-x$
344	30960	$-344x$
$4x$	$360x$	$-4x^2$

$R = (90-x)(344+4x)$

$R = 30960 + 16x - 4x^2$

$R' = 16 - 8x$

$0 = 16 - 8x$

$8x = 16$

$x = 2$

$344 + 4(2) = 352$

Excellent

$\boxed{\$352.00}$

9. Find all critical points of $f(x) = \sin^2 x$ and classify them as maxima, minima, or neither.

Take derivative:

$$f'(x) = 2(\sin x)' \cdot \cos x$$

Set to 0:

$$0 = \underbrace{2\sin x}_{0 \text{ when } x = n\cdot\pi \text{ for } n\in\mathbb{Z}} \cdot \underbrace{\cos x}_{0 \text{ when } x = \frac{\pi}{2} + n\cdot\pi \text{ for } n\in\mathbb{Z}.}$$

Find second derivative:

$$f''(x) = 2\cos x \cdot \cos x + 2\sin x \cdot -\sin x = 2\cos^2 x - 2\sin^2 x$$

Classify:

Where $x = n\pi$, $f''(x) = 2(\pm 1)^2 - 2(0)^2 = 2$

$\therefore$ Those are minima.

Where $x = \frac{\pi}{2} + n\pi$, $f''(x) = 2(0)^2 - 2(\pm 1)^2 = -2$

$\therefore$ Those are maxima.

So the function has maxima at $x = \frac{\pi}{2} + n\pi$ (for n an integer)
and minima at $x = n\pi$ (for n an integer)

10. Find the points of inflection and the intervals on which $y = \dfrac{\ln x}{x}$ is concave up and concave down.

$$y' = \frac{\left(\frac{1}{x}\right)(x) - (\ln x)(1)}{x^2} = \frac{1 - \ln x}{x^2}$$

So there's a critical point where $1 - \ln x = 0$, so $\ln x = 1$ or $x = e$.

$$y'' = \frac{\left(\frac{-1}{x}\right)(x^2) - (1 - \ln x)(2x)}{(x^2)^2} = \frac{-x - 2x + 2x\ln x}{x^4} = \frac{-3 + 2\ln x}{x^3}$$

So $y'' = 0$ when $-3 + 2\ln x = 0$, so $\ln x = 3/2$ or $x = e^{3/2}$ as our inflection point.
Test points show it's concave down left of there and concave up to the right.
The domain doesn't include 0 or any negatives, so we have

Concave down on $(0, e^{3/2})$
Concave up on $(e^{3/2}, \infty)$.