Exam 4 Calc 1 12/11/2015

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the definite integral.

2. Evaluate $\int_1^3 \frac{1}{x} dx$.

3. If you use a left-hand sum with $n = 4$ subdivisions to approximate $\int_1^3 \frac{1}{x}\,dx$, what (to at least 4 decimal places) are:

$\Delta x =$

$c_1 =$

$c_2 =$

$c_3 =$

$c_4 =$

$f(c_1) =$

$f(c_2) =$

$f(c_3) =$

$f(c_4) =$

$\sum_{i=1}^{4} f(c_i) \cdot \Delta x =$

4. Find the area of the region bounded between $y = x^2$ and $y = 4x$.

5. Evaluate $\int \frac{1}{(2x-8)^4}\,dx$.

6. Suppose $\int_0^2 f(x)\,dx = 3$, $\int_2^5 f(x)\,dx = 7$, $\int_0^2 g(x)\,dx = 2$, and $\int_2^5 g(x)\,dx = 15$.

   a) Evaluate $\int_0^2 3g(x)\,dx$.

   b) Evaluate $\int_0^5 f(x)\,dx$.

   c) Evaluate $\int_2^5 \left[f(x)+6\right]dx$.

7. a) Evaluate $\dfrac{d}{dx}\int_0^x \dfrac{1}{1+t^5}\,dt$.

   b) Evaluate $\dfrac{d}{dx}\int_0^{x^2} \dfrac{1}{1+t^5}\,dt$.

8. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap. This Calculus stuff is pretty rough. So, like, sometimes I get negative numbers when I do the definite integrate things, right? So the answer in the back of the book is pretty much always just what I got but with the negative taken off, right? So I heard it's like that because you sometimes get things upside down, like with the bottom thing first or whatever, right? So do you always just take the negative sign off?"

   Help Biff by explaining as clearly as you can whether his reasoning holds, or if there are limitations

9. Evaluate $\int \frac{x\,dx}{a+bx}$.

[Hint: if it's hard with the constants *a* and *b* in there, warm up with 2 and 3 in those slots.]

10. Consider the curves $y = \sin x$ and $y = \cos x$, along with the lines $x = 0$ and $x = 2\pi$. What is the total area of the region bounded by these curves on the interval $[0, 2\pi]$?

Extra Credit (5 points possible):
Pick two points (let one have $x = a$ and the other have $x = b$) on the first-quadrant portion of the hyperbola $y = 1/x$. Find the area bounded by the line between those two points and the hyperbola.