**Examlet 1** **Graph Theory** **10/1/15**

Do question 1 and pick four of the remaining questions for grading (mark them clearly or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. Define the following terms:
    - complement of a graph
    - tree
    - connected graph
    - cycle (in a graph)
    - spanning tree

2. For which values of $n$ does a graph with degree sequence $(7,7,6,5,5,3,3,n)$ (note that the degreee sequence is not necessarily in proper decreasing order) exist?

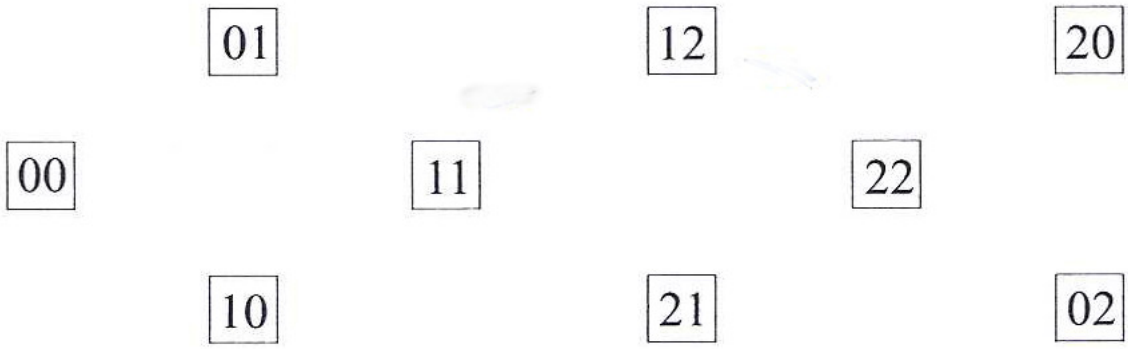
3. Show that all 6-regular graphs with eight vertices are isomoprhic to each other.

4. Show that if a graph contains at least one odd cycle, then the graph is not bipartite.

5. How many vertices of degree 1 can exist in a tree with $n$ vertices? Support your answer without using any previous results.

6. Suppose that a graph $G$ has a unique edge $e$ of maximal weight. Can a minimal spanning tree contain $e$? Provide an example or prove it can't happen.

7. Put in the edges, with labels, in this 9-vertex state diagram.



8. What can you say about Hamilton paths and cycles in bipartite graphs? Justify your claims without using any previous results.