Examlet 2 **Graph Theory** **11/5/15**

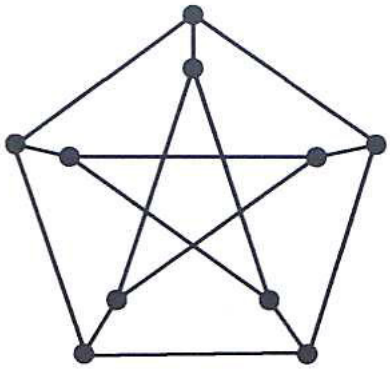
Do questions 1 through 3 and pick two of the remaining (lettered) questions for grading (check boxes of those you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. Define the following terms:
    - planar graph
    - independent vertex set
    - proper vertex coloring
    - vertex coloring number χ
    - vertex cut set

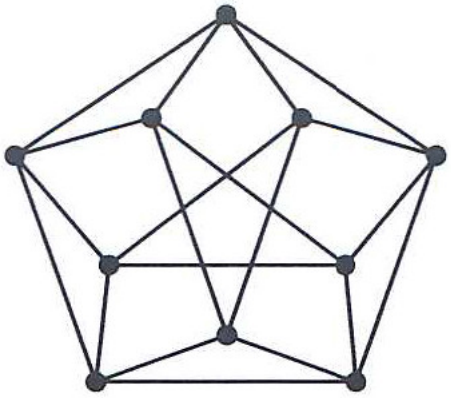
2. State and prove Euler's Formula.

3. Show that $\mathbf{K}_5$ is non-planar.

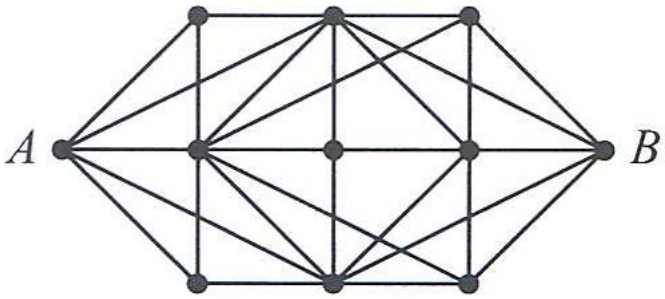
□ A. Show that the graph below is non-planar.



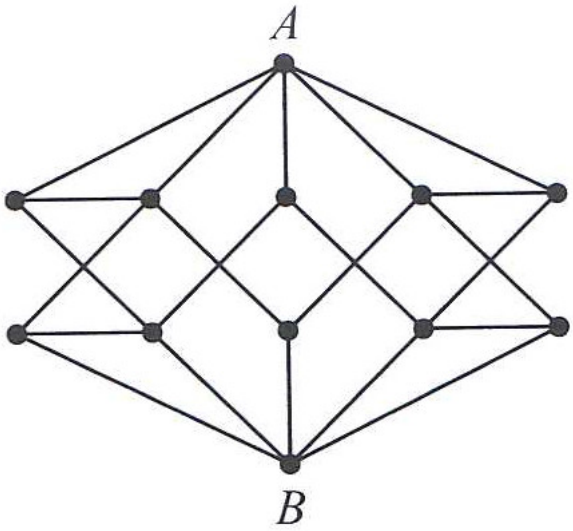
□ B. What is the vertex coloring number of the graph shown below, and why?



□ C. What is the maximum number of internally disjoint paths from A to B in the graph shown below, and why?



□ D. What is the maximum number of edge-disjoint paths from A to B in the graph shown below, and why?



□ E. i. What is the maximum number of independent vertices in the graph shown below, and why?

ii. What is the maximum number of independent edges in the graph shown below, and why?