Exam 3   Calc 1   10/28/2016

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. a) What is $(\sinh x)'$?

    b) What is $(\cosh x)'$?

    c) What is $(\tanh x)'$?

2. What is $\left(\dfrac{\ln x}{x}\right)'$?

3. Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 - x}$.

4. What is $\left(x \tan^{-1} x \right)'$?

5. What is $\left(\sin^{-1}\left(x^2 \right) \right)'$?

6. A sample of tritium-3 decayed to 94.5% of its original amount after a year. How long would it take the sample to decay to 10% of its original amount?

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "I think calculus is only for geniuses! At first I thought the elope-it-all rule thing was really easy, but on our exam I guess I really messed up. I did it like this, and the grader gave me zero. That's so unfair!"

$$\lim_{x \to \left(\frac{\pi}{2}\right)^+} \frac{\cos x}{1-\sin x} \overset{l'h}{=} \lim_{x \to \left(\frac{\pi}{2}\right)^+} \frac{-\sin x}{0-\cos x} \overset{l'h}{=} \lim_{x \to \left(\frac{\pi}{2}\right)^+} \frac{-\cos x}{\sin x} = \frac{0}{1}$$

Explain clearly to Bunny what she should understand about using L'Hôpital's Rule here.

8. Why is $\dfrac{d}{dx}(\sinh^{-1} x) = \dfrac{1}{\sqrt{1+x^2}}$?

9. In 1696 the first calculus textbook was published, with an example finding the limit of

$$y = \frac{\sqrt{2a^3 x - x^4} - a\sqrt[3]{aax}}{a - \sqrt[4]{ax^3}}$$

as x approaches a. Evaluate this limit.

10. We showed on the last exam why the Power Rule for derivatives works for all natural number exponents, like x^5 (by using the Product Rule). Show that it follows that the Power Rule for derivatives also works for roots like $\sqrt[5]{x}$.

Extra Credit (5 points possible): What is the derivative of x^x?