Exam 4 Calc 1 11/18/2016

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Find all critical numbers of $f(x) = 2x^3 - 3x^2 - 36x$.

2. Find all inflection points of $f(x) = 2x^3 - 3x^2 - 36x$.

3. The function $f(t)=\dfrac{1}{0.2+e^{-t}}$ is being used to model a learning curve for students in a study. What is the largest interval on which f is concave down? [Hint: $f'(t)=\dfrac{25e^t}{\left(e^t+5\right)^2}$ and $f''(t)=\dfrac{-25e^t(e^t-5)}{\left(e^t+5\right)^3}$]

4. Find the absolute minimum and maximum values of $f(t)=t\sqrt{4-t^2}$ on $[-1,2]$.

5. Use one iteration of Newton's Method (so find x_1) with an initial approximation of $x_0 = 2$ to approximate a solution to $0 = 2x^3 - 5x^2 + 2x - 7$.

6. If $f''(x) = -2 + 12x - 12x^2$, $f(0) = 6$, and $f'(0) = 12$, what is $f(x)$?

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this calculus stuff is tough. I was understanding the derivative stuff pretty well, with the increasing and decreasing and stuff, right? But then the professor started talking about the second derivative, so like if increasing is increasing or decreasing, I guess? But I don't get it, and he was talking about, like, caves or something? Like cave up and cave down? But his accent is really tough, and I don't think I get it."

   Explain clearly to Biff what the second derivative tells us about the shape of a graph.

8. [B&C] Squares with sides of length x are cut out of each corner of a rectangular piece of cardboard measuring 5 ft by 8 ft. The resulting piece of cardboard is then folded into a box without a lid. Find the volume of the largest box that can be formed in this way.

9. A newly developed electric vehicle has energy consumption (measured in Watt-hours per mile driven) given by $E(v) = 0.2v^2 + 1000/v$, where v is the velocity of the vehicle (in miles per hour). At what velocity (exactly) will the car's energy consumption for a trip be lowest?

10. A can of pumpkin pi filling is being designed to hold a volume of 200π cm^3. What radius and height will minimize the surface area (bottom, top, and sides) of the can?

[Tips: For a circle, Area = πr^2 and Circumference = $2\pi r$. For a cylinder, Volume = $\pi r^2 h$.]

Extra Credit (5 points possible): Towns at the points (0,0), (4,0), and (2,7) are to be joined by a road network. What is the shortest collection of roads that will connect these three points?