Exam 1 Calc 3 9/23/2016

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the formal definition of the partial derivative of a function $f(x, y)$ with respect to x.

2. Suppose that w is a function of x, y, and z, each of which is a function of t. Write the Chain Rule formula for $\dfrac{dw}{dt}$. Make very clear which derivatives are partials.

3. Find an equation for the plane tangent to $f(x, y) = 4x^2 - y^2$ at the point $(1, -3, -5)$.

4. Show that $\lim_{(x,y)\to(0,0)} \frac{xy - x^2}{x^2 + y^2}$ does not exist.

5. Let $f(x, y) = x^2y^3$, and let $P = (1/6, 3)$

a) At the point P, find the directional derivative in the direction of $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

b) In which direction is the directional derivative greatest at P, and how large is the directional derivative in that direction?

6. Show that for any vectors $\vec{a}$ and $\vec{b}$, the vector $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Crap, this Calc 3 stuff is killing me. I understood pretty good about derivatives, because they're pretty much like Calc 1, and that's okay, right? But crap, now this second derivative test just has too much crap going on for me to get it straight. And especially the stuff where D is zero, you know? I figured one out with help from my buddy where D was 0, but it was really a max. Isn't it really that 0 always means it's a max, and they're just not telling us `cause they want to make it as hard as they can?"

   Explain clearly to Biff whether he can count on all critical points where D is 0 being maxima, or not, and why.

8. Find the minimum value of $f(x, y) = 2x + 3y$ subject to the constraint $x^2 + y^2 = 25$.

9. Find and classify all critical points of $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

10. Jon wants to 3d-print a solid that transitions smoothly from the paraboloid $z = 9 - x^2 - y^2$ between $z = 0$ and $z = 8$, into a portion of a cone above $z = 8$. The cone will need to match the radius and slope of the paraboloid at the height where they join. The cone will have an equation of the form $z = d - m\sqrt{x^2 + y^2}$. What are the appropriate values for d and m?

Extra Credit (5 points possible):
Find the point in the first quadrant on the curve $y = x + x^{-1}$ closest to the origin.