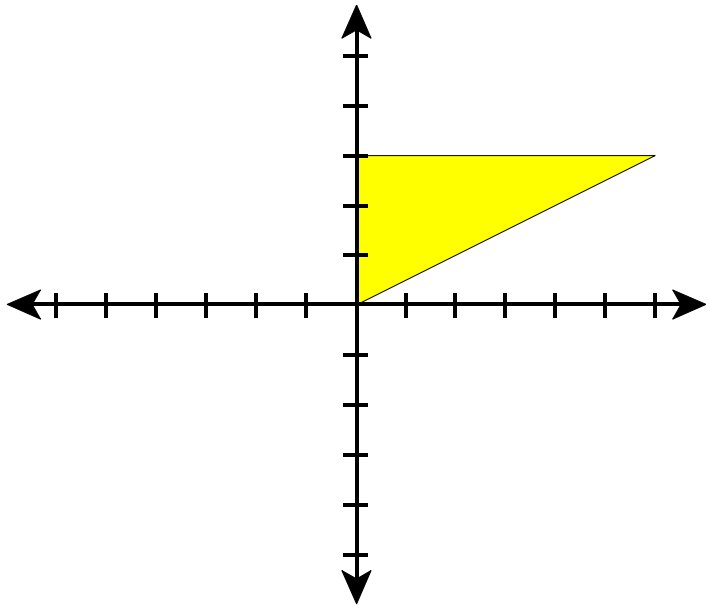
Exam 2 Calc 3 10/21/2016

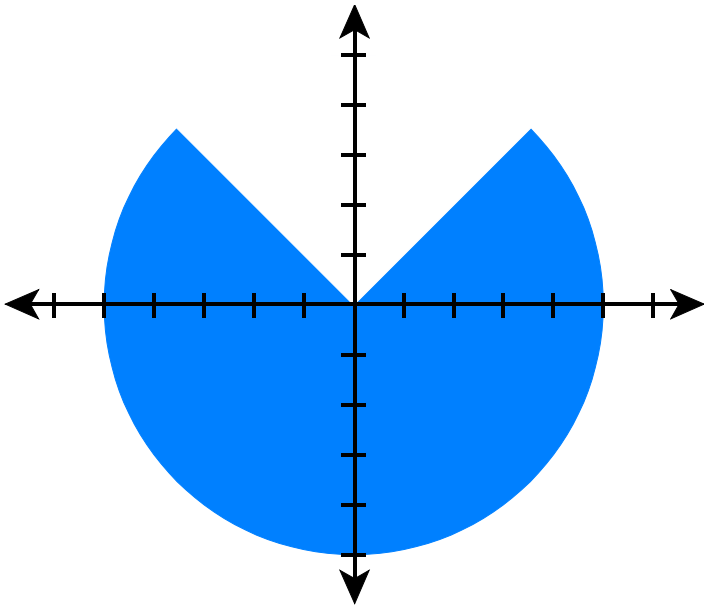
Each problem is worth 10 points. For full credit provide complete justification for your answers. All integrals should be set up in terms of a single coordinate system, i.e., if you use cylindrical your integral should involve no x or y, etc.

1. Write a double Riemann sum for $\iint_R f\,dA$, where R is the rectangle with vertices (0,0), (6,0), (6,4), (0,4) using midpoints with $n = m = 2$ subdivisions.

2. Set up an iterated integral for the volume below $z = f(x, y)$ and above the xy-plane on the region R pictured below:



3. Set up an iterated integral for the total mass of a plate shaped like the region shown below, with density $\rho(x, y) = k$.



4. Set up an iterated integral for the volume of a really nice greenhouse with a roof shaped like $y^2 + z^2 = 100$ and base a rectangle $R = [0,20] \times [-6,6]$.

5. Evaluate $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} 3\,dz\,dy\,dx$.

6. The waiting time for certain support phone calls includes an initial hold period of length x and a subsequent hold period of length y once the call is directed to an appropriate specialist, for the random variables X and Y with joint density function

$$p(x,y) = \begin{cases} \frac{1}{15} e^{-x/3 - y/5} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Set up an iterated integral for the probability that total hold time for a call is over 6 minutes.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, this Calc 3 stuff is just too much. I can work out integrals pretty well, but some of the shapes are just so confusing! And then it seems like you're supposed to know all this stuff about symmetrical stuff, and it's really confusing. There was this one question on our exam about, like, what regions $\iint_D x\,dA$ is zero for because they're symmetrical. How could I even figure that out?"

   Explain clearly to Bunny some instances of regions for which $\iint_D x\,dA$ would be zero, and why.

8. Find the Jacobian for the transformation from rectangular to spherical coordinates:

$x = \rho \sin \phi \cos \theta$

$y = \rho \sin \phi \sin \theta$

$z = \rho \cos \phi$

9. A hole with radius 2cm is drilled vertically through a sphere with radius 10cm. The sphere is made of copper, which has a density of 8.96 g/cm^3. Set up an iterated integral for the mass of the resulting solid.

10. Set up iterated integrals for the z coordinate of the center of mass of a pyramid with uniform density, height 1, and a square base with side length 2.

Extra Credit (5 points possible):

The cone $z = 9 - 3\sqrt{x^2 + y^2}$ is cut by the plane $z = 6 - 2x$. Find the volume of the region above the plane but below the cone.