Problem Set 1 **Calculus 3** **Due 10/19/16**

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 3 points for correct and clearly justified answers, and spelling your name correctly on your submission is worth 1 point.

1. [Based on Rogawski & Adams 3rd §15.5 Example 7] Without proper maintenance, the times to failure (in months) of two sensors in an aircraft are random variables X and Y with joint density function

$$p(x,y) = \begin{cases} \frac{1}{864}e^{-x/24-y/36} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   What is the probability that neither sensor functions after 1 year?

   It sets up almost identically to Example 7 in the text, so

$$\int_0^{12} \int_0^{12} \frac{1}{864} e^{-x/24-y/36}\, dy\, dx$$

   Mathematica says that works out to about 0.0111536.

2. [Based on Rogawski & Adams 3rd §15.5 #51] The lifetime (in months) of two components in a certain device are random variables X and Y with joint density function

$$p(x,y) = \begin{cases} \frac{1}{9216}(48-2x-y) & \text{for } x \geq 0, y \geq 0, 2x+y \leq 48 \\ 0 & \text{otherwise} \end{cases}$$

   Calculate the probability that both components function at least 6 months without failing.

   It sets up almost identically to the homework problem in the text, and the top view is very similar to the one given there – note how the 18 there becomes a 21 here, so we have

$$\int_6^{21} \int_6^{48-2x} \frac{1}{9216}\,(48-2x-y)\; dy\, dx$$

   which *Mathematica* says is $\frac{125}{512} \approx 0.244141$.

3. The waiting time for certain support phone calls includes an intial hold period of length x and a subsequent hold period of length y once the call is directed to an approiate specialist, for the random variables X and Y with joint density function

$$p(x,y) = \begin{cases} \frac{1}{15}e^{-x/3-y/5} & \text{for } x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that the total hold time for such a call is over 10 minutes?

Look at the top view. The easiest way to approach it is to find the integral representing calls less than 10 minutes, so where $x + y \leq 10$, and then conclude that the probability we want is the rest, or 1 minus that share. So the integral we set up is

$$\int_0^{10} \int_0^{10-x} \frac{1}{15} e^{-x/3-y/5}\, dy\, dx$$

Mathematica says that works out to about 0.715173, so the probability of a call lasting longer than 10 minutes is $1 - 0.715173 = 0.284827$.