Exam 1 **Real Analysis 1** **9/30/16**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of a function $f(x)$ converging as x approaches a.

2. a) State the definition of an accumulation point.

b) Give an example of a set which is infinite but has no accumulation points.

3. a) State the definition of a Cauchy sequence.

b) Give an example of a sequence which is Cauchy.

4. a) Give an example of a set which is infinite and bounded, and which has a maximum element.

   b) Give an example of a set which is infinite and bounded, and which does not have a maximum element.

5. a) State the Bolzano-Weierstrass Theorem for Sets.

b) State the Triangle Inequality.

6. a) State the Sum Rule for limits of sequences.

b) State the Quotient Rule for limits of sequences.

7. State and prove the Product Rule for limits of products of functions as x approaches a.

8. If a sequence $\{a_n\}$ diverges to $+\infty$ and $a_n \leq b_n$ for all $n \geq n_1$, then the sequence $\{b_n\}$ must also diverge to $+\infty$.

9. a) Prove or give a counterexample: If a sequence $\{a_n\}$ is convergent, then it is eventually increasing and bounded.

b) Prove or give a counterexample: If a sequence $\{a_n\}$ is eventually increasing and bounded, then it is convergent.

10. If $f : D \to \mathbb{R}$ and $\lim_{x \to a} f(x)$ exists, then f is bounded on some set D_1, with $D_1 \subseteq D$ and a an accumulation point of D_1.