

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let $A, B \in \mathbb{R}$. Show that $\forall \epsilon > 0, |A - B| < \epsilon \Rightarrow A = B$.
2. Show that if a sequence is convergent, then it is bounded.
3. Suppose that $\{a_n\}_{n=1}^{\infty}$ is a convergent sequence. Is $\{a_{2n}\}_{n=1}^{\infty}$ convergent as well?
4. Suppose that $\{a_n\}$ and $\{b_n\}$ are sequences such that $a_n = b_n$ for all even values of $n \in \mathbb{N}$, and suppose $\{a_n\}$ converges. Does $\{b_n\}$ converge as well?
5. The sequence $\{a_n\}$ converges to 0 if and only if the sequence $\{|a_n|\}$ converges to 0.
6. The sequence $\{a_n\}$ converges to A if and only if the sequence $\{|a_n|\}$ converges to $|A|$.
7. Let $\{a_n\}$ be defined by $\frac{3n}{2n+1}$ for $n \in \mathbb{N}$. Show that this sequence converges
8. If $\{a_n\}$ and $\{b_n\}$ differ from each other in only a finite number of terms, then both sequences converge to the same value or they both diverge. [Kosmala 2.1.8(b)]