Problem Set 4 **Real Analysis 1** **Due 9/26/16**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Every Cauchy sequence is bounded. [No fair using the CCC!]

2. If $\{a_n\}$ is a Cauchy sequence and $S = \{a_n | n \in \mathbb{N}\}$ is finite, then $\{a_n\}$ is constant from some point on.

3. Let s_0 be an accumulation point of S. Then any neighborhood of s_0 contains at least one point of S different from s_0 iff any neighborhood of s_0 contains infinitely many points of S.

4. Suppose $\{a_n\}$ and $\{b_n\}$ both oscillate. Then $\{a_n \cdot b_n\}$ and $\{a_n + b_n\}$ oscillate.

5. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where f and g are functions with domain D. Prove (directly from the definition) that $\lim_{x\to\infty}[f(x) \cdot g(x)] = A \cdot B$.

6. If a limit exists for f as x approaches a, then that limit is unique.

7. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where f and g are functions with domain D. If $\forall x \in D, f(x) < g(x)$ then $A < B$.

8. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where f and g are functions with domain D. If $\forall x \in D, f(x) \leq g(x)$ then $A \leq B$.

9. If $f : D \to \mathbb{R}$ and $\lim f(x)_{x\to a}$ exists, then f is bounded.

10. If $f : D \to \mathbb{R}$ and $\lim f(x)_{x\to a}$ exists, then f is bounded on some set D_1, with $D_1 \subseteq D$.