Exam 3   Calculus 3   12/1/2017

Each problem is worth 10 points. Show adequate justification for full credit. Please circle all answers and keep your work as legible as possible.

1. Parametrize and give bounds for a line segment from $(-3, 0)$ to $(1, 2)$.

2. Let $\mathbf{F}(x, y, z) = \langle 2xz, z^2 + 1, x^2 + 2yz \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the circle $x^2 + y^2 = 4$ with counterclockwise orientation.

3. Let $\mathbf{F}(x, y, z) = \langle 3y, 2x, 5z \rangle$, and let S be the sphere with radius 1 centered at (a, b, c). Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

4. Compute $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x, y) = xy\,\vec{i} - y\,\vec{j}$ and with C a line segment from (1,2) to (3,–1).

5. Let C be the positively oriented square with vertices (0,0), (4,0), (4,3), (0,3). Evaluate the line integral $\int_C 9y^2x\,dx + x^2y\,dy$.

6. Prove that if $\mathbf{F}(x,y,z)$ is a vector field whose component functions have continuous second-order partial derivatives, then div(curl $\mathbf{F}$) = 0. Make it clear how the requirement that the partials be continuous is important.

7. Biff is a calc 3 student at a large state university and he's having some trouble. Biff says "Well crap, I thought I was good at math until Calc 3. I'm pretty good at getting answers, you know? But now we gotta say why there's no answers, which is pretty stupid. This one on our test was to show why there's no potential function for $\langle -y, x\rangle$, which is crazy, 'cause probably you just haven't looked hard enough, right? I mean, doesn't there gotta be an answer?"

   Explain as clearly as possible to Biff what you can say about the existence of a potential function for his vector field, and why.

8. Let **F** be the vector field $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + 2\,\mathbf{k}$. Let S be the portion of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 5$, oriented away from the z-axis. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

9. Let $\mathbf{F}(x, y, z) = y\,\mathbf{i} + z\,\mathbf{j} + x\,\mathbf{k}$, and let S be the hemisphere $x^2 + y^2 + z^2 = 1$, $y \geq 0$, oriented in the direction of the positive y-axis. Evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.

10. Let $\mathbf{F}(x, y) = \langle 5y, 5x \rangle$. Find the radii and centers of all circles C such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 9$.

Extra Credit (5 points possible):

Let $\mathbf{F}(x, y) = \langle 5y,\ 5x + x^2 \rangle$. Find the radii and centers of all circles C such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 9$.