Quiz 4 Calculus 3 11/13/17

Each problem is worth 5 points. Clear and complete justification is required for full credit.

1. Compute $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x,y) = xy\,\vec{i} - y\,\vec{j}$ and with C a line segment from (1,2) to (4,–3).

$\vec{F} = \langle xy, -y \rangle$

$\vec{r} = \langle 1+3t, 2-5t \rangle$

$0 \le t \le 1$

$\vec{F}(\vec{r}(t)) = \langle (1+3t)(2-5t), -2+5t \rangle$

$= \langle 2+6t-5t-15t^2, -2+5t \rangle$

$= \langle 2+t-15t^2, 5t-2 \rangle$

$\vec{r}\,'(t) = \langle 3, -5 \rangle$

$\int_0^1 \langle 3, -5 \rangle \cdot \langle 2+t-15t^2, 5t-2 \rangle \, dt$

$\int_0^1 6+3t-45t^2-25t+10 \, dt$

$\int_0^1 16-22t-45t^2 \, dt$

$16t-11t^2-15t^3 \Big|_0^1$

$16-11-15$

$16-26$ Great

$\boxed{-10}$

Yes, there's a second problem on the back!

2. Compute $\int_C \vec{F} \cdot d\vec{r}$ for the vector field $\vec{F}(x,y) = \langle 5x^4y^2, 2x^5y \rangle$ and with C the counterclockwise arc of a circle beginning at (0,0) and ending at $\left(\sqrt{2}, \sqrt{2}\right)$.

$\vec{F}(x,y) = \langle 5x^4y^2, 2x^5y \rangle$

$f_{xy} = 10x^4y \quad f_{yx} = 10x^4y$

We can use Fun. Thm. of Line. Int.

$f(x,y) = x^5y^2$

$\left[x^5y^2\right]_{(0,0)}^{(\sqrt{2},\sqrt{2})}$

$= (\sqrt{2})^5 \cdot (\sqrt{2})^2 - 0$

$= 2(2)^{5/2}$

$= 2\sqrt{32}$

$= 2\sqrt{16 \cdot 2}$

$= 2 \cdot 4 \cdot \sqrt{2}$

$= 8\sqrt{2}$

Nice!