Exam 1b Calc 1 9/13/2018

Each problem is worth 10 points. For full credit provide complete justification for your answers.

Use the graph of $g(x)$ at the bottom of the page for problems 1 and 2:

1. Find the following limits:

   a) $\lim_{x \to -1^-} g(x) = \underline{1}$

   b) $\lim_{x \to -1^+} g(x) = \underline{2}$

   c) $\lim_{x \to -1} g(x) = \underline{\text{DNE}}$ because the limits from the right and left don't match and the point -1 is not defined

   d) $\lim_{x \to 5^+} g(x) = \underline{1}$

   Great

   e) $\lim_{x \to 3^-} g(x) = \underline{0}$

   f) $\lim_{x \to 3} g(x) = \underline{0}$ limits from the left equal the ones from the right

2. For which values of x does the function fail to be continuous?

Dis Continuous at:

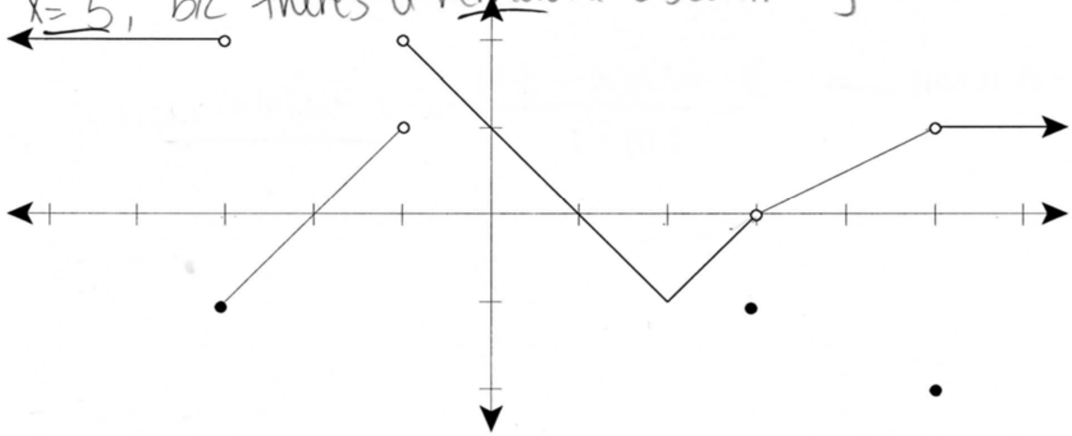
X=-3, b/c g(x) doesn't approach the same limit

X=-1, b/c g(x) doesn't approach the same limit (jump discontinuity)

X= 3, b/c there's a removable discontinuity

Good

X= 5, b/c there's a removable discontinuity



3. Evaluate $\lim_{x\to 3}\frac{x^2-9}{x-3}$ $= \lim_{x\to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$ considering $x \neq 3$

$= \lim_{x\to 3} (x+3) = (3)+3 = \boxed{6}$

Checking: $f(x) = \frac{x^2-9}{x-3}$

Excellent!

$f(2.99) = 5.99$ ✓

4. [Stewart 1.3] If a rock is thrown upwards on the planet Mars with a velocity of 10m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$. Find the average velocity of the rock over the following time intervals:

   a) [1,2] $\frac{12.56-8.14}{2-1} = 4.42$ m/sec

   b) [1,1.5] $\frac{10.815-8.14}{1.5-1} = 5.35$ m/sec

   Excellent!

   c) [1,1.1] $\frac{8.7494-8.14}{1.1-1} = 6.094$ m/sec

   d) [1,1.01] $\frac{8.202614-8.14}{1.01-1} = 6.2614$ m/sec

t	t(h)
0	0
1	8.14
1.01	8.202614
1.1	8.7494
1.5	10.815
2	12.56

5. Provide a justification on each line for the corresponding equality:

$$\lim_{x\to 2}\frac{x^2-9}{x-3}=\frac{\lim_{x\to 2}\left(x^2-9\right)}{\lim_{x\to 2}\left(x-3\right)}$$ Quotient Law for Limits

$$=\frac{\lim_{x\to 2}x^2-\lim_{x\to 2}9}{\lim_{x\to 2}x-\lim_{x\to 2}3}$$ Difference Law for Limits

$$=\frac{\left(\lim_{x\to 2}x\right)^2-\lim_{x\to 2}9}{\lim_{x\to 2}x-\lim_{x\to 2}3}$$ Power Law for Limits

$$=\frac{2^2-\lim_{x\to 2}9}{2-\lim_{x\to 2}3} \quad \begin{matrix} y=9 \\ y=3 \end{matrix}$$ Law X for Limits

$$=\frac{2^2-9}{2-3}$$ Constant Law for Limits

$$=\frac{5}{1}=5$$ Great

6. a) Evaluate $\lim_{x\to\infty} \frac{2x^2+7}{(x-1)(x^2+x)}$. $= \lim_{x\to\infty} \frac{2x^2+7}{x^3+\cancel{x^2}-\cancel{x^2}-x}$

$$= \lim_{x\to\infty} \frac{2x^2+7}{x^3-x} \cdot \frac{\left(\frac{1}{x^3}\right)}{\left(\frac{1}{x^3}\right)}$$

$$= \lim_{x\to\infty} \frac{\frac{2x^2}{x^3}\to 0 + \frac{7}{x^3}\to 0}{\frac{x^3}{x^3}\to 1 - \frac{x}{x^3}\to 0} = \frac{0}{1} = \boxed{0}$$

Checking: $f(1000) = \frac{2(1000)^2+7}{(999)(1000^2+1000)} = .002000..$ ✓

b) Evaluate $\lim_{x\to 1^-} \frac{2x^2+7}{(x-1)(x^2+x)}$.

$$= \lim_{x\to 1^-} \frac{2x^2+7}{x^3-x} \quad \text{(see work above)}$$

$2(.999)^2+7 = 8.996$

$.999^3 - .999 = -.0019$

Here I can see that the denominator will become a tiny negative number as $x\to 1^-$ and the numerator will approach 9 $(2(1)^2+7)$. This means the fraction will be an arbitrarily large negative number $\left(\frac{(+)}{(-)} = (-)\right)$.

So. $\boxed{\lim_{x\to 1^-} \frac{2x^2+7}{(x-1)(x^2+x)} = -\infty}$

Excellent!

Checking: $-\frac{8.996}{.0019} = -4734.73$ ✓

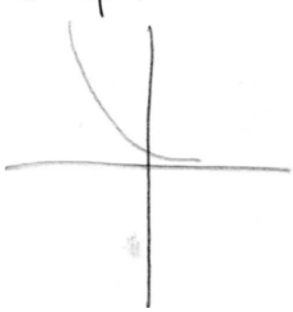
7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. Our Calc class I guess makes a pretty big deal about limits, even though I never saw those in high school at all, which is pretty unfair if you ask me. But so I was trying to figure them out, and the TA said something about it being like speed limits, I guess? So there was this function e^{-x} that we were s'posed to say the limit of, but it looked to me like the graph was coming down towards 0, right? But with a speed limit, like, you're not allowed to go above it, right? So is it okay for 0 to be the limit even if it goes down to that, instead of getting up to it?"

Help Biff by explaining as clearly as you can the answer to his questions.

A limit isn't defined on whether or not it has to approach it's limit from the top or the bottom. In Biff's case, the TA told him it was like a speed limit that you can't go over and though you can get close, you can't reach the actual limit. I think the TA's problem is that they didn't explain that it works the same way going downward. There are such things as negative graphs so using a separate real world application such as population could help him understand the negative side of limits.

Excellent!

Graph



8. Evaluate $\lim_{x\to\infty}\left(\sqrt{4x^2+x}-2x\right)$. Be sure to provide good justification for your conclusion.

$$\lim_{x\to\infty} \sqrt{4x^2+x}-2x \cdot \frac{\left(\sqrt{4x^2+x}+2x\right)}{\left(\sqrt{4x^2+x}+2x\right)}$$

← this is just a funny-looking 1 that I picked because cool things happen when you multiply.

$$= \lim_{x\to\infty} \frac{4x^2+x-(2x)^2}{\sqrt{4x^2+x}+2x}$$

← Diff. of Squares

An epic war between good and evil in the numerator leaves x standing alone so the highest power is now 1. I multiply by another funny-looking 1.

$$\lim_{x\to\infty} \frac{\cancel{4x^2}+x-\cancel{4x^2}}{\sqrt{4x^2+x}+2x} \cdot \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\frac{x}{x}}{\sqrt{\frac{4x^2}{x^2}+\frac{x}{x^2}}+\frac{2x}{x}} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

because $\frac{1}{x}=\frac{1}{\sqrt{x^2}}$

Outstanding!

Checking: $f(10000)=\sqrt{4(10000)^2+10000}-2(10,000)$

$= .249998$ ✓

9. Evaluate $\lim_{h \to 0} \dfrac{(h-1)^3 + 1}{h}$.

$$\lim_{h \to 0} \frac{(h-1)(h-1)(h-1)+1}{h}$$

$$\lim_{h \to 0} \frac{(h^2 - \overset{-2h}{h - h} + 1)(h-1)+1}{h}$$

$$\lim_{h \to 0} \frac{h^3 - 2h^2 + h - h^2 + 2h \cancel{-1+1}}{h}$$

algebra

$$\lim_{h \to 0} \frac{h^3 - 3h^2 + 3h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(h^2 - 3h + 3)}{\cancel{h}}$$

mystery law ☺

$$\lim_{h \to 0} h^2 - 3h + 3$$

$$\lim_{h \to 0} 0^2 - 3(0) + 3$$

Law X for limits

$$\lim_{h \to 0} = 3$$

Excellent!

10. Jon is trying to produce a function of the form $f(x) = \begin{cases} x^2 & \text{for } x \le 2 \\ 4x + c & \text{for } x > 2 \end{cases}$ for some value of c that makes the graph continuous. Is this possible? If so, what should the value of c be?

If it's continuous, the limits from the right + left are equal and $\lim_{x \to a} f(x) = f(a)$ at every point.

$\therefore$ $x^2 = 4x + c$ at $x = 2$

$(2)^2 = 4(2) + c$

$\boxed{c = -4}$ $\longrightarrow$ check: ① $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = 4$

② $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (4x - 4) = 4$

Well done

and $f(2) = \lim_{x \to 2} f(x)$.

So YES, it's possible.